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ONE, TWO, THREE… A DISCUSSION ON THE GENERATION OF NUMBERS IN PLATO'S *PARMENIDES*

Abstract

One of the questions regarding the *Parmenides* is whether Plato was committed to any of the arguments developed in the second part of the dialogue. This paper argues for considering at least one of the arguments from the second part of the *Parmenides,* namely the argument of the generation of numbers, as being platonically genuine. I argue that the argument at 142b-144b, which discusses the generation of numbers, is not deployed for the sake of dialectical argumentation alone, but it rather demonstrates key platonic features, such as the use of the greatest kinds and the generation principle. The connection between the argument for the generation of numbers and Plato's philosophy of mathematics is strengthened by the exploration of a possible reference in Aristotle's *Metaphysics* A6. Taken as a genuine platonic theory, the argument could have significant impact on how we understand Plato's philosophy of mathematics in particular, and the ontology of the late dialogues in general – that numbers can be reduced to more basic entities, i.e the greatest kinds, in a way similar to the role the greatest kinds are assigned in the *Sophist*.

Keywords: Plato, the *Parmenides*, Aristotle, mathematics, generation of numbers, *one*, *two*, *three*, multiplication, *one*, *being*, *difference*, the greatest kinds, even, odd

This paper considers an aspect of Plato's view on numbers which is almost unexplored by scholars, namely the argument for the generation of numbers from the *Parmenides* 142b-144b. In the first Section, I identify Aristotle's references to Plato's philosophy of mathematics in an attempt to isolate possible interpretations of the argument for the generation of numbers. I also provide an outline of the argument for the generation of numbers by reconstructing step by step the progression of thought as found in the *Parmenides* 142b5-143a2 and 143a4-144a5. Section Two develops possible links between Aristotle's *Metaphysics* A6 and the *Parmenides*, through an exploration of current scholarship. Finally, in Section Three I return to the argument for the generation of numbers providing an analysis of the key features of its construction. In the light of this reading I stress the need for a reevaluation of the argument for the generation of numbers.

Section One - Overview of Aristotle's Testimonies

In several dialogues, Plato showed an intense interest in the definitions, elements of mathematics, and philosophy of mathematics, he dealt with numbers, arithmetic, and geometry, and paid a vivid attention to mathematical methods.[1] But how exactly one should understand the ontology of numbers and the place of mathematics in his philosophy, or if Plato contributed to the development of mathematics on his own remains a rather ambiguous tasks for both ancient philosophers (e.g. Aristotle) and modern readers.[2] The dialogues do not give us a coherent view on how Plato understood the ontology of mathematical objects, but provide us with rich references to mathematics. Accordingly, the dialogues testify Aristotle's claims that Plato was immersed in the problematic ontology of mathematical objects. However, several of Aristotle's testimonies regarding Plato's philosophy of mathematics are in many regards conflicting and confusing, and complicate substantially any attempt at making sense of how Plato understood the ontology of mathematical objects. Aristotle attributed at least seven partly contradictory views to Plato. Accordingly, for Plato:

a) numbers are forms (*Met.* 1073a17-22, 1090a16-17),
b) numbers are intermediary objects between forms and physical particulars (*Met.* 987b14-17, 1028b19-21, 1059b5-14, etc.),
c) individual instances exist by participation to numbers (*Met.* 987b12),
d) numbers are the product of the one and the dyad (*Met.* 987b22-35, 1092a23-24),
e) numbers are generated out of the dyad, except those which are prime (Met. 987b23-988a1)
f) form numbers are only up to ten (*Phys.* 206b33, *Met.* 1084a10, 25),
g) forms are numbers (*Met.* 991b9, 1081a12, 1083a-1084a, *De Anima* 404b24-25).

All these partially conflicting and competitive testimonies point out that Plato's philosophy of mathematics was from the very beginning a controversial issue. Plato's dialogues give straight support for some of the Aristotelian claims, especially for (a), (b), (c). The number-form theory (a) could fit the views from the *Phaedo* (101b9-c9, 103-106), while the assessment that numbers are intermediaries between forms and things (b) could find some grounds in the *Republic* (509d-511a), depending on how one interprets the divided line (epistemologically or ontologically), and in the *Philebus* (56c-59d). The presumption that things exist by participation to numbers (c) could be traced in the *Timaeus*, where, unlike any of the Aristotelian conceptions, physics and mathematics are related. *Timaeus* exhibits this theory, since mathematics is an essential feature of the physical world, although it is not evident how the mathematical objects from the *Timaeus* can be linked with (a) and (b). However, in the *Timaeus*, Plato does not construct the physical particulars through numbers, but through geometrical objects. Physical bodies are composed of particular geometrical entities. At their turn the structure of these entities is determined by two types of right-angled triangles: isosceles (45°/45°/90°) or scalene, (30°/60°/90°). Thus the triangles are the ultimate "atoms" of the matter.

The supposition that Plato reduced numbers to one and the indefinite dyad (d) is excessively – and almost exclusively – defended by the Tübingen School as the real system of Plato, and it relies minimally on platonic texts, and mainly on Aristotle's and post-Aristotelian testimonies. That Plato had thought of form-numbers only up to ten (f) and that forms are numbers (g) seems to be a peculiarity of Aristotle's interpretation, and it completely lacks any reference in platonic dialogues.

Despite all these possibilities, the main scholarly controversy in the field is almost exclusively on a) versus b) – whether, according to Aristotle, Plato understood mathematical objects as forms[3] (P. Shorey[4] and H. Cherniss,[5] or, more recently, P. Pritchard,[6] or W. Tait[7]) or as intermediaries between forms and things (A. Wedberg,[8] or M. Burnyeat[9]). The grounds for these two main conflicting views on Plato's understanding of mathematical objects rely heavily on Aristotle's testimonies, which most favored the intermediary position. However, the two views seem to be irreconcilable, and scholars argue for one or the other position; one must add that scholars who support a) or b) assume that Plato had a fixed theory, of the intermediary or of the number-forms, which basically is unchanged from the *Phaedo* and the *Republic* to the later dialogues.

The statement e) in which Aristotle criticizes Plato that he does not generate prime numbers, even if the rest are generated, is very precise and seems to be alien to the dialogues and to the conventional way of seeing Plato as a Platonist regarding numbers. If there is a place in the Platonic corpus where one should look for something that could resemble Aristotle's testimony, it is in the second part of the *Parmenides* (142b-144b), where a generative process for obtaining numbers is presented, an argument which, with few exceptions, is ignored by scholarship. The whole argument, divided in two parts (142b5-143a2, and 143a4-144a5), aims to prove that the *one is multiple*, and, accordingly, there is a *generation of numbers*.

An outline of the argument of the generation of numbers as developed in the *Parmenides* 142b-144b is offered below:

(142b1,5) Parmenides returns to the hypothesis from the beginning (ἐξ ἀρχῆς):

(142b) "if one is, can it *be*, but not partake of being?"

I. (142b-c) [if one is, is both one and being]

1. If the/a one *is*,
2. then the one partakes (μετέχειν) of being,
3. the one is not the same as being (as being of the one),
4. „is" signifies something other than "one,"
5.> *one partakes of being.*

II. (142d-143a)

1. the one is a whole, being and one are its parts,
2. oneness is not absent from the being(-ness) part, and being(-ness) is not absent from the oneness part;
3. each of the two parts possesses oneness and being, the part is composed of at least two parts, endlessly, since oneness always possesses being and being always possesses oneness.
4. (by necessity) it always comes to be two, it is never one,
5.> *the one is infinitely many (unlimited and multitude).*

III. (143a-b) [the introduction of difference]

1. *one* is not *being*.
2. *one* has a share in *being*.
3. therefore one and *its* being are *different*.

4. *one is not different from being in virtue of its oneness.*
5. *being is not different from one in virtue of being itself.*
6. > therefore the difference of one and being is due to difference.
6. > therefore there is difference and it is distinct from one and being.

IV. (143c-144b) [The argument from one admitting all numbers, from "member of a pair" and "two," Plato constructs the whole number system]

7. (143c3) if we have three distinct entities we can pick out pairs (τινε)[10] (say being and difference, or being and unity, or unity and difference).

8. (143c4) a pair is rightly called 'both'[11] (ἀμφοτέρω) ["x", "y" = "both (x,y)"].

9. (143d2) what is called both is two *(*δύο*)* [a "pair" is identified as a set with two members corresponding to the cardinal number two].

10. (143d2-3, 4-5) each of the two is one (δύο ἦτον > ἓν εἶναι).

11. (143d7) one added to any sort of pair is three (τρία γίγνεται) [a set off three members corresponding to the cardinal number three].

(if two & three, then all the numbers)

12. (143d8-9) three is odd (Τρία... περιττὰ), and two even (δύο ἄρτια),

13. (143d9-e2) if there are two (δυοῖν), there must be twice (δίς), since two is twice one (τῷ τε δύο τὸ δὶς ἓν)

14. (143e1-2) if three (τριῶν), also thrice (τρίς), since three is thrice one (τῷ τρία τὸ τρὶς ἕν).

15. (143e3) from 13. there must be "twice two" (δύο δὶς).

16. (143e3) from 14. there must be "thrice three" (τρία τρὶς).

17. (143e5) from 13) &14) there must be twice three (τρία δὶς) and thrice two (δύο τρίς)

18. (143e7) there *will* be even times even (Ἄρτιά ἀρτιάκις), odd times odd (περιττὰ περιττάκις), odd times even (ἄρτια περιττάκις), and even times odd (περιττὰ ἀρτιάκις).

19. (144a3) "and if it is so, do you think there is any number left that does not necessarily exist?"

20. (144a4) if one is, there must also be number (Εἰ ἄρα ἔστιν ἕν, ἀνάγκη καὶ ἀριθμὸν εἶναι).

The above can be synthetized as follows:

a. In the first part of the argument (142b5-143a2), Parmenides argues, and his opponent, Aristotle[12], accepts, that if one *is*, it means that one *has* being, therefore *one* and *being* are *two* distinct entities; and the *is*-ness of *one* can be separated. The parts of one - i.e. *one* and *being* - are at their turn *one* and *being* as well, and so on, *ad infinitum*. The division by two is in the following way:

One Being
One Being; One Being
One Being; One Being; One Being; One Being;
δύ‘ ἀεὶ γιγνόμενον μηδέποτε ἓν εἶναι
(always becomes two and never one, 143.a.1)

O B - 2
O B + O B - 4
O B; O B + O B; O B - 8
O B; O B; O B; O B + O B; O B; O B; O B – 16
O B; O B; O B; O B; O B; O B; O B; O B +
O B; O B; O B; O B; O B; O B; O B; O B - 32

It is unclear whether one should take the division of *one* into two entities as a proper division (as the word itself indicates) or as a progressive multiplication. The multiplication/division implies either:[13]

a) the same number series of multiples of two: 2, 4, 8, 16, 32, etc.
b) divisions by two: ½, ¼,1⁄8, 1/16, etc.

Whether it is a principle of division or of multiplication (or both, as in the cellular division), one can see that the ontological power of division of *one* into *one* and *being* produces only powers of duality.

b. The second part of the argument (143a4-144a5) restates the same division of one, this time introducing, alongside one and being, *difference* – as *the* principle of differentiation between one and being. Thus one is not different from being because of its oneness (of being one), nor because of its being (of being being), but the difference between them is by virtue of difference and otherness (τῷ ἑτέρῳ τε καὶ ἄλλῳ ἕτερα ἀλλήλων, 143b7). Next, the argument switches to the issue of picking up pairs (being and difference, or being and unity, or unity and difference), and concludes that one added to any sort of pair is three. And from this: (143e7) there will be even times even (Ἄρτιά ἀρτιάκις), odd times odd (περιττὰ περιττάκις), odd times even (ἄρτια περιττάκις), and even times odd (περιττὰ ἀρτιάκις); thus all the numbers (144a4). It is argued that all numbers are generated from these three entities through a necessary process of multiplication. Thus, contrary to what one would expect, namely to derive numbers through addition, Plato used multiplication instead. The rest of the numbers after two and three are products: 2x2, 3x3, 2x3, and so on. The multiplication process leaves out prime numbers (144a4).

Considering Plato's conception on mathematical objects, that numbers are either intermediates or forms, this argument raises several questions. A central one for platonic scholarship is whether Aristotle refers precisely to this argument when he criticizes Plato's generation of numbers.

Section Two - Aristotle and the *Parmenides*

Aristotle claims on several occasions that for Plato numbers are generated, but it is only in one place that he explicitly insists that Plato's process of generation leaves out prime numbers. In *Metaphysics* A6 (987b29-988a1), Aristotle states about Plato:

> His divergence from the Pythagoreans in making the One and the numbers separate from things, and his introduction of the Forms, were due to his inquiries in the region of definitory formulae (for the earlier thinkers had no tincture of dialectic), and his making the other entity besides the One a dyad was due to the belief that the numbers, except those which were prime, could be neatly produced out of the dyad as out of a plastic material.[14]

The passage, together with the whole chapter A6, is famously problematic. In A6, Aristotle associates quite naturally Plato's philosophy with that of the Pythagoreans[15] showing at the same time where Plato's philosophy differs from that of the Pythagoreans. An obvious idea which occurs from Aristotle's confident testimony is that, as with the Pythagoreans, a central point in Plato's philosophy was to give an account on numbers. At the first view, the generation of numbers from the *Parmenides*[16] cannot be the argument Aristotle had in mind, since, one may argue, "the generation of numbers does not seem to have been a concern of Plato.[17] Furthermore, Aristotle claims that for Plato numbers "could be neatly produced out of the *dyad* as out of some plastic material", but the *dyad* is apparently missing from the *Parmenides*. David Ross thinks that "the *Parmenides* does not help us, for there is no question there of the indefinite dyad."[18] Also, in a note to his translation of the *Metaphysics*, he claims that in the *Parmenides* "primes are not there excepted" and "nothing in the works of Plato corresponds exactly to what Aristotle says here."[19] However Ross does not say more about why he thinks that primes are not missing from the *Parmenides'* generative process. The claim is quite surprising, and Ross does not provide any further clues. The clue is given only in his edition to the Greek text: "the numbers, including 2, are produced by the ordinary processes of addition and multiplication from 1."[20] However, for Ross, Aristotle's account in A 6 "is not quite accurate" since in N 1091a9-12 "it is only 2 and its powers that could be neatly produced out of the 1 and the indefinite dyad."[21] Indeed, despite of what Aristotle claims in A6, at N3 1091a10 he remarks that Platonists "cannot in any way generate numbers other than those got from 1 by doubling",[22] consequently the dyad generates multiples of two[23] (powers of two), a series of 2, 4, 8, 16, 32, etc.

Ross' observation that in the *Parmenides* "there is no question… of the indefinite dyad" is too easily assumed since the first part of the argument generates a series of dualities, even if numbers are not explicitly mentioned. Julia Annas argues for the contrary: the specification of the *Parmenides* argument at 142b5-143a2 "resembles the working of the indefinite two", while at 143a4-144a5 "indicates a process which is reminiscent of the way one works as a principle."[24] Thus Annas considers that one could read and appropriate Aristotle's remark on Plato's dyad (not only the instnance from A6, but also those from M and N) in the light of the *Parmenides* 142b-144b. According to A. E. Taylor, to whose opinion I submit, it is this very argument from the *Parmenides* that Aristotle had in mind in his "perversion of Plato's theory of numbers" with the one and 'indeterminate Duality'.[25] If Aristotle had referred to the *Parmenides* this would prove mainly that the argument on generation of numbers argument was an issue which Plato considered as an actual possibility, and we should consider it accordingly.

Section Three – Analysis of the argument of the generation of numbers

The validity of the argument was questioned by Malcolm Schofield, who pointed out its fallacious inferences.[26] For the purpose of this paper the question of the validity of the argument is of secondary importance since I don't aim to discuss if Plato's argumentation is sound, but I am considering the possible intentions, and philosophical conjectures and inferences that can be drawn thereupon.

3.1 - The Origin of the Number Two

In addition to the example of cellular division of the pair one-being into identical cells of the same type (142d-143a), several nuances of duality are used in the second stage of the argument. It is specified that if we have three distinct entities we can pick out pairs (for example, being and difference, or being and unity, or unity and difference). A pair is rightly called 'both' (ἀμφοτέρω) ["x", "y" = "both (x, y)"], and what is called both is two (δύο). Plato takes δύο (which matches the set with two members and the cardinal number two[27]) as a consequence and derivation of ἄμφω. Accordingly the duality of ἄμφω resists to a reduction to the ordinal or cardinal feature of the number two, and naturally the cardinality of δύο is posterior to the *pair* relation of ἀμφοτέρω.

This is a very peculiar method of inferring that there is a set with two members. One can assume a conceptual distinctiveness of *duality* because there are many pairs which are not countable, and are a *unity* in themselves; for this type of pair it is difficult to define which one is the first in an ordinal way, as in the instance of two eyes, two arms, two legs, two ears – the most common case of dual numeral in the ancient Greek language. Two eyes are a pair, since one cannot decide which of the eyes comes first when counting. In this case, nobody would actually count the two eyes; their quantification is almost *a priori.* The parity of the two eyes is a given; it is a duality, and counting them individually is merely a process of division; its number, as a pair, is prior to the counting of two units. One must add that the one-being pair is not quite like the pair of two eyes, ears, etc. since we don't have a pair of identical things, but a pair of two different entities: one and being. Nevertheless, this ontological pair relation is more powerful than the pair relation of similar and identical things, which can be conceived independently of each other.

Picking up pairs is a certain mathematical operation. But how can one understand it? Is it a mental operation? Is it a metaphysical principle? A linguistic determination given by the dual of the Greek language? Plato's aim to emphasize the priority of ἄμφω compared to that of δύο, if they are taken as dyad and two, could be behind Aristotle's critique against the theory that the dyad is prior to number two, as it is to be found in *Metaphysics* A9. Here Aristotle develops an argument in which he emphasizes that it is actually impossible to assume that the cardinality of two is posterior to that of the dyad, as Plato thinks. According to Aristotle: "in general the arguments for the Forms destroy the things for whose existence we are more anxious than for the existence of the Ideas; for *it follows that not the dyad but number is first* (my emphasis), i.e. that the relative is prior to the absolute" (990b18-20). The tension is between the indefinite dyad and the forms, which stand here for numbers (number-forms). Aristotle's argument is as follows:

i) the dyad is prior to the forms (of numbers);
ii) the form number 2 is posterior to the dyad;
iii) but the dyad already contains the cardinality of two;
iiii) therefore a contradiction.

In spite of the coherence of the argument, it is difficult to think that Plato would have accepted proposition iii) – to have the indefinite dyad as an instance of δύο.[28] If Aristotle's counterargument aimed the *Parmenides* (as Taylor and Annas assumes), it was not a strong argument, since in the *Parmenides* the cardinality of two is obtained with difficulty at a later stage of the argument.

Plato's insistence on picking up a certain pair (τινε) which "is correctly called 'both'"[29] (ὀρθῶς ἔχει καλεῖσθαι ἀμφοτέρω) before saying that they are two shows us that he didn't consider obtaining δύο as a simple and univocal procedure (ex. 1+1). The procedure of obtaining *two* is the following: we take a τινε (a pair of two eyes, two legs etc.) that we divide into each member (e.g. ἑκάτερος, which will be the opposite of ἀμφότερος), and thus we have independent members from which we can have *two*. Thus the *pair relation* comes first, and after that their numerosity, namely that there are two things. The *twoness* of the *one* and *being* is an instance of ἀμφοτέρω (*duality*), and not of the cardinality of δύο (number *two*), in the sense that the duality refers to pairs, while two itself refers to only any two things.[30] Ross is right in saying that: "Aristotle is not quite fair in assuming that the indefinite dyad is an ordinary member of the class of 2's".[31] The cardinality of the dyad is only an Aristotelian reading, and a sophism.[32]

Plato's elaborated process for obtaining *two* is quite complicated and, to some extent, it might seem gratuitous. There should be a reason why such a waste of concepts around duality, which is not obvious from the

Parmenides alone. Plato could have been also aware of the kind of critique later formulated by Aristotle, so he constructed such a detour argument. The derivation of two from pairs goes straight against *Metaphysics* A9, 990b18-20. *Two is not obtained by the addition of one to one; rather, each member of two is one.* Thus, the reason for which Plato does not derivate cardinal number two directly from *counting* two entities (as one and being, or being and difference), but from a *pair* could have been to avoid the kind of critique that Aristotle later had in mind.

3.2 - The Origin of the Number Three

Contrary to Aristotle's assertion, the argument from the *Parmenides* doesn't stop at δύο, *one* is obtained (ἓν εἶναι) from two (δύο ἦτον), which now stands as a unity for calculations, which is used only once properly (143d7) for generating a set with three members (τρία γίγνεται). Thus the unity for calculations (which is not number one) is obtained via δύο, and not from the beginning, from initial ἕν. This is another peculiar procedure since *one* was already mentioned and used at the start of the argument (142b-c). It could be that Plato thinks that *one* cannot be *one* at all in the beginning (A = ~A) as a unity for calculation, "since it always proves to be two, it must never be one" (143a). *One* cannot exist as a unitary entity, but as a member of the one-being pair. Instead, the first hypothesis (137c–142a) analyses *one* by itself, *one* which is *one* (A = A).

The new *one* (which is obtained after *two*!) would be enough for generating numbers, like 1+1+1…. and so on. Yet Plato does not follow what would be for us the obvious way; but offers a surprise. Even if he starts with three entities - "difference is not the same as oneness or being" (143b6-7) referring thus to three entities -, he later develops a numerosity with three members, thus the cardinal three is obtained by the addition of a unity to any pair (143d7). Why was it necessary, in order to obtain three, of such a complex turn (stressing the *one* which is added to any pair)? Is the way of obtaining three from 2+1 and not from 1+1+1 a metaphysical necessity? Are not the first three entities (one, being, and difference) enough for obtaining number three or threeness? One can venture that obtaining number three only from counting one, being and difference would not emphasize the oddness as it does 2+1 (as part of formula 2k+1).

At *Phaedo*, 105c, Plato explicitly maintains that oneness is the *sine qua non* condition for an odd number to be odd: "if asked the presence of what in a number makes it odd, I will not say oddness but oneness." Three is the first odd number, if we exclude one as being odd – a controversial issue for ancient mathematicians. The first remark that Plato makes, after generating three, is to say that "three is odd." In the economy of the multiplicative generation, compared with the rest of the numbers, which are generated by multiplication, threeness is *a product of addition* of one to two. Only this number is obtained by addition, or for this number the process of adding 1 is emphasized, since it is unclear if addition is used or could be used for the rest of the numbers.

3.3 - Multiplication *versus* addition operations

Contrary to what one would expect, namely to derive numbers through addition, Plato used multiplication instead. The rest of the numbers after two and three are products of multiplication: 2x2, 3x3, 2x3, and so on (even if their products are not called numbers, it is obvious that it is about numbers as results of these products). The first occurrence of the term number is in the conclusion: "Then if that is so, do you think there is any number that need not be?" - "In no way at all." - "Therefore, if one is, there must also be number" (144a4-5). Plato starts with three entities – *one, being,* and *difference* – and he infers that from them one can have all the numbers. There could be more entities, as for example, *motion* and *rest*, but only three entities are the necessary ingredients to have all the numbers. It is a question if *one, being* and *difference* are *the* entities for obtaining numerosity, or there could be any three entities. I am inclined to think that they have an ontological priority similar to that of the composition and the generation of the world soul (*Timaeus* 34c-35b) where same, being, and difference are the basic ingredients. Having been given the three ingredients of the *Parmenides*, the numbers two and three come –even if in an apparently complicated manner – as a natural consequence, and the multiplicity evolves from them, since the rest of the numbers are expressions of the first two and three. Probably that Plato thinks that the multiplication operation is inherent to generation of two and three (which stay as proto-numbers), and implicitly of their correlatives - evenness and oddness, since Plato consider that if there are two, there must be twice, since two is twice one, and if three, also thrice, since three is thrice one.[33]

The shortcoming of obtaining numbers only through multiplication is that primes remain unreached, since they are not multiplies of two or of three.[34] Thus through multiplication alone one cannot obtain the complete number series; nevertheless at the end of the argument it is claimed the opposite. Why did Plato put so much explicit emphasis on multiplication, and omitted addition? We usually define number series through addition, as *n+1*. Was this not the case for Plato too? It seems to be counterintuitive to generate numbers through multiplication and modern theories of numbers and philosophy of mathematics provide no correspondent at all.

A ground for leaving addition aside may be that multiplication provides a better understanding of numerosity since each number can be reduced to (prime) factors, and it is easier to reduce numbers to basic factors of 2 and 3 than to 1 (e.g. 6=1+1+1+1+1+1 versus, *simpliciter*, 6=3x2 or 2x3). Only in this way the primary even and primary odd are proven necessary for number series. Each number, except the primes, is reduced to factorial operation, of the first even and first odd number. The priority of evenness and oddness is not alien to *Phaedo*, where alongside good itself of just itself, even itself and odd itself are given as examples of forms.

But what are the odd and even? Are these qualities? Do they behave like forms? The classification of numbers into even and odd is an important feature of Plato's generation of numbers, and the relation between *three* and *odd*, and *two* and *even* could be analogous to that between *fire* and *hot*.[35] Any definition they may be given, one can see that through the identification or classification of the odd and even, one can spot subsequently all the numbers. [36] One of Plato's aims within the argument is to arrive at the first odd and even numbers, and thus to proceed to the generation, consistent to his commitment (as in the *Republic* VII 524d, *Theaetetus* 198a, *Gorgias* 453e) that the knowledge of numbers is the knowledge of odd and even.

For Plato, and for some Pythagoreans, the even and odd are not proprieties of numbers, but rather numbers are proprieties and derivations of even and odd. Such a classification is natural in Greek mathematics, and, indeed, as Thomas Heath noticed, "Euclid's classification does not go much beyond this." [37] Does Plato think that through the prediction that there will be even times even, odd times odd, odd times even, and even times odd his procedure is exhaustive and the primes are somehow generated too? Here are some hypotheses: The addition is used together with the multiplication process, and thus 5 could be case of 4+1, similar to that of 3=2+1; or, if we could take 1 as an odd number, primes could be a case of odd times odd, and thus 5 would be 5x1; or primes could be a subgroup of the odd group. Another option, to which Aristotle submits, is that primes remain ungenerated and Plato thought about them as being ungenerated since one cannot reduce them to a factorial procedure, and primes would be the real form-numbers. All these hypotheses have their limits, and it's not my purpose to discuss them in this paper. I would mention only that regarding the last hypotheses, that of Aristotle, the problem would be that the first prime numbers are generated: 2 generated from the pair relation, and 3 from 2+1. Nevertheless, any solution given to *the primes problem*, must take into account that Plato's emphasis on multiplication (which cannot exclude the possibility that Plato thought about it as a repeated addition) aims to present the building blocks of numbers (2 and 3), while for the primes there will be always a question who are their building blocks (they themselves?).

3.4 - Ontological or Chronological Generation

In the *Phaedo* (101b9-c9) we find the claim that numbers exist only in virtue of the forms of numbers, which have the same ontological status as the form of beauty or courage.[38] Here Plato asserts that *two* participates in the form of *twoness*, and *one* in the form of *oneness*, and we might be inclined to think accordingly with regards to all numbers. Perhaps Plato refers to the fact that there is more than one number 2 (and only one form of *two*), as, for example, in '2+2=4'. If so, number 2 itself cannot be a form, only just one of the many individual instances that participate in the form of *twoness*. It does not mean that the form of two (the *twoness*) is composed by two (form) entities, but that there is only one unique and uncompounded form for every two things.[39] It is not a particular quality (or an adjective) of two, or three, or four, etc. Each number (as a form) is a unit and each numerosity, i.e. what is counted (not as a form), is composed of units. The form of the number is a simple unity and only one entity; the concrete numeral is a plurality, composed by entities. That might entail that numeral 2 consists of 1+1, but *twoness*, as a form number, is not composed of oneness plus oneness. In other words, numbers *are not generated through addition*, and are abstract entities, an infinite number of abstract and eternal entities.

Given the view that Plato is more or less a Platonist regarding numbers, how should one conceive a possible *generation of numbers*?[40] Is it a proper generation in action, firstly, from 2 obtaining 1, and from 2+1 obtaining 3, and secondly, from 2 and 3 by multiplication all the numbers? Could it mean that there was maybe a moment when numbers did not exist? Should one think about the generative process as being one in time, in which each new number receives *existence* and appears from nowhere, as it would have not existed before, or it's a matter of reducing numbers to an axiomatic frame?

The argument for the generation of numbers should remind us of the Pythagorean understanding of the generation of numbers. In this regard, W. K. C. Guthrie points out that the Pythagoreans didn't perform a sharp distinction between "logical and chronological priority."[41] However, there are scholars who think the opposite, namely that "the generation of numbers was regarded by the early Pythagoreans as an actual physical operation occurring in space and time, and the basic cosmogonical process was identified with the generation of numbers from an initial unit, the Monad."[42] Aristotle (*Met.* 1091a 12-29) also took number generation as a process in time, even if the Academy had not endorsed such an interpretation.[43] Annas reproaches to Aristotle's literalism the possibility that the Academy may have not distinguished "between a historical account and a logical analysis"[44]. Thus one should not think of the generation of numbers *per se*, but of the existence of a number series, an existence proof and a classification of numbers.[45] Robert Turnbull considers that, though *gignomai* is similar with the term used by Plato with the meaning of 'coming to be', involving a temporal meaning, "here is a standard one of Greek mathematicians for the generation of various mathematical series."[46] At *Parmenides* 153a-b, Plato is speaking of the number one as being older than the numbers which follow after it, which are younger:

> So, the least thing first; and this is the one. Isn't that so? (…) But that which has come to be first, I take it, has come to be earlier, and the others later; and things that have come to be later are younger than what has come to be earlier. Thus the others would be younger than the one, and the one older than they.

This understanding of number as being in time is unusual and can hardly be accommodated with a timeless reading of number generation. One would suppose that qualifications such as 'earlier' or 'later' which are attached to numbers of the series do not necessarily refer to their chronological organization.

The notion of generation itself is misleading and in order to properly understand it, one should consider that there is a *hierarchical priority* of entities as *twoness* and *threeness* on the one hand, and the *multiplicity* on the other hand. Even if there is no generation in time, numbers are generated by a previous *existence* of two (*duoin*) and three (*trion*) and by pairs of the three kinds.

One may ask whether the generation of numbers would be a philosophical problem for Plato, since we think usually that Plato could have numbers conceived of as forms or as intermediates. There is also the debate if Plato kept the theory of forms after the *Parmenides*. Currently, scholars speak about a (1) *Revisionist* position (he rejected the theory of Forms in the late dialogues) and a (2) *Unitarian* one (Plato is consistent in all his dialogues with the theory of forms). [47] The two interpretations exclude each other, especially when applied to dialogues such as the *Parmenides*, the *Theaetetus*, and the *Sophist*.

I think that the generation process presented in the *Parmenides* should be read through the lens of the generation of the soul from the *Timaeus* (35c-36c), and the *Sophist*. It might be that, with the *Parmenides*, the generation process marks a turning point in Plato's philosophy and that in late dialogues Plato did indeed revised his theory of forms into a *meta-theory of forms and numbers* (3), in which more primitive Forms/Logical Entities (the 'greatest kinds' of the *Sophist*: *being, sameness, difference*) are at the core of a new ontology, and new philosophical possibilities are explored: in the *Sophist* a new theory of language is formulated, in the *Parmenides* a theory for the generation of numbers, and in the *Timaeus* a theory of the constitution of the soul.

Plato was immersed in the problem of generation, and could have been considering a scheme aimed to support the idea of the generation of numbers (comparable to the Pythagorean generation of numbers); as he often spoke about different types of generations: generation of the world, generation of the soul, generation of the numbers, in which, most probably, the generation process stands for a logical analysis and a description of the structure of what is 'generated'. The reassessment of Plato's philosophy into a meta-theory is coherent with the Revisionist theory (1), and should not contradict the Unitarian theory (2), since, as I think, for looking and constructing an axiomatic system for the theory of forms (and not only), Plato goes beyond his early view on forms (as it is presented and developed in the *Phaedo*, the *Republic*, the *Parmenides*), without necessarily refuting them. The solution that I advance is not merely one of a compromise between (1) and (2), but an attempt to hint towards the reconstruction of a possible ontology of the late dialogues in which unclear, but fundamental passages – such as the generation of numbers or the generation of the soul – could be harmonized with both the theory of forms and that of the greatest kinds in Plato's late dialogues.

3.5. - The *Parmenides* evaluates Parmenides

The second argument from the *Parmenides* could be essential to advance the view, stressed by the Tübingen School, that Plato had an "unwritten doctrine" which is not developed in the written dialogues. The appropriation of the dyad with the division by two at 142b5-143a2 is a vast enterprise and it is beyond the purpose of the article, which insists mainly on the duality of one and the consequences thereupon. Moreover, my aim is not to insist on the implications for a possible oral teaching of Plato, since it is contradictory to reconstruct an allegedly exclusive oral teaching from a written one. I try thus to avoid as much as possible the methodology of the Tübingen School, when interpreting the passage from the *Parmenides*. What is important in the economy of my paper – that Plato's own philosophical position can be recognized in the second argument of the second part of the *Parmenides* – is to emphasize that Aristotle was aware of such perpetual division of *one* into the duality of *one* and *being*, and that he overlaps this division with a function that he attaches to the dyad.

The difference between a possible Tübingen interpretation of the passage and that which is expressed in this paper is that a Tübingen reading makes difficult to follow the development of the greatest kinds introduced in the *Sophist*, while my reading gives to the *Sophist* a more coherent meaning: the greatest kinds of the *Sophist* are thus an elaboration of an ontological project which is only partially elaborated in the *Parmenides'* second argument.[48] It's not necessary to read the possible relation between the *Parmenides* and A6 exclusively through the lens of the "unwritten doctrine", but that one can have a more nuanced reading of 142b-144b, especially 143a-b, and 143c-143e2, in which several divisions and dualities are at work in order to point to a generative process. The duality of one goes straight against the unwritten doctrine in which the One and the indefinite dyad are the two independent, opposing principles. I take the duality of one as opposing the monolithic feature of one from the first hypothesis (137c–142a). In contrast to the unwritten doctrine, where the dyad is an independent principle from one, the duality (i.e. the dyad) comes from the relation between the *one* and (its) being, and could rather point to the poem of Parmenides.

Plato develops a hypothesis-argument immediately after the first hypothesis that concluded with the claim that nothing can be said about, and predicated to the *one*. Both hypotheses are mirroring each other (some of the negation of the first one become affirmation in the second one) and they are meaningful if we consider them as part of a larger philosophical agenda. Even if it is a dialectical exercise or not, the second part of the *Parmenides* is relevant for a reconstruction of Plato's philosophy. As Ross put it, treating the 'second part' as "primarily a gymnastic exercise does not exclude the possibility that in the course of it Plato may hit on positive ideas which will fructify in his later thought."[49]

Most of the scholars, such as Runciman[50] or Ryle[51] think that it is Plato's form which is under examination in the second part of the Parmenides, and, implicitly, in the first hypothesis. However, I think that there are good reasons to consider that what Plato examines in the first hypothesis, and the whole second part, is exactly Parmenides' view. Cherniss,[52] Guthrie[53] or Turnbull[54] argue also in this regard, that one should keep in mind Parmenides' poem and that the subject matter of the second part of the *Parmenides* is the Parmenidean concept of *one-being*. The second hypothesis could be read as a correction of the non-predicative standing of the Parmenidean being,[55] while sampling a few of the greatest kinds developed later in the *Sophist*. My claim on the genuineness of the second hypothesis, and its link to the *Sophist*, is even more legitimate if one considers that Plato's account here is part of a larger project which is developed in the *Sophist*, and partially in the *Timaeus*. Thus the question if the conjectures around the derivation of numbers from allegedly some of the greatest kinds have any implication at all to Plato's philosophy could be reformulated as: how a Plato of the *Sophist* would understand the *structure* and the *ontology* of numbers? The most reasonable answer is to look again at the *Parmenides'* generation of numbers.

Conclusion

This paper contributes to expanding the range of interpretative approaches to Plato's philosophy of mathematics, raising some not so obvious questions (such as, for example, why 3 is a derivation of 2+1 and not of 1+1+1) around the argument of the generation of numbers, showing that one may consider Plato's ontology of numbers in a different frame than that given by the current discussion - that Plato either had numbers as forms or as intermediates (which are conceived as ungenerated in both cases). Gesturing towards the resemblance between Aristotle's testimonies and the *Parmenides*, Section One of this paper establishes that a more detailed exegesis of the *Parmenides* is warranted. On foot of this I proceeded to carry out this exegesis by providing a schematic presentation of the argument for the generation of numbers. After setting the outline of the argument for the generation of numbers I move to Section Two, in which these possible traces are identified in the *Metaphysics* A6. The argument for the generation of numbers, set out in Section One, raises several problematic issues which are explored in detail in Section Three. Different interpretative possibilities are proposed in relation to the origin of number two, the origin of number three, the choice of multiplication versus addition operations, the nature of generation, the subject matter of the *Parmenides'* second part. I advance the idea that Plato intentionally made a loop to substantiate the idea of generation of cardinal number *two* from a duality in order to respond to a possible critique, as later offered by Aristotle, that the cardinal number two is inherent to duality. I argue that in the *Parmenides* (142b-144b), with the help of three entities (*one*, *being*, *difference*), which resemble the greatest kinds of the *Sophist* (to be developed in further studies), Plato constructs possibly platonic arguments for the derivation of the first even and the first odd numbers, from which the rest of numbers are generated.

NOTES

1 Scholars argue that Plato's interested in mathematics had a great impact on the development of the discipline in the Academy and elsewhere. A passage from Proclus' commentary on the first book of Euclid's *Elements*, entitled "Catalogue of Geometers", written probably by Aristotle's student, Eudemus, records that Plato was "greatly advanced in mathematics in general and geometry on particular because of his zeal for these studies. It is well known that his writings are thickly sprinkled with mathematical terms and that he everywhere tries to arouse admiration for mathematics among students of philosophy", Proclus, *Commentary on the First Book of Euclid's* Elements, transl. Glenn R. Morrow (Princeton: University Press, 1970), 54. On the other hand, some argue just the opposite: "Plato's role has been widely exaggerated. His own direct contributions to mathematical knowledge were obviously nil", or "Plato 'directed' research is fortunately not borne out of the facts", Otto Neugebauer, *The Exact Sciences in Antiquity* (New York: Dover, 1969), 152. A similar skepticism is shared by Ian Muller: "It is very unlikely that Plato made substantive contributions to mathematics; indeed, many of the more specifically mathematical passages in his works have no clear and correct interpretation, and many of them can be read as the half-understandings of an enthusiastic spectator", Ian Mueller, "Mathematics and the Divine in Plato", in Teun Koetsier and Luc Bergmans, (eds.), *Mathematics and the Divine* (Amsterdam: Elsevier Science, 2005), 99-121, 101. See also Harold Cherniss, "Plato as Mathematician", *Review of Metaphysics* 4, no. 3 (1951): 395–425.

2 In antiquity, in the third century A.D., Diogene Laertius, *Lives* (3.24) attributed to Plato the discovery of geometrical analysis. Three centuries later, *Anonymous Prolegomena Philosophiae Platonicae* (5.32-35) claimed the same thing. However, these testimonies are doubtful to the modern scholar. See, for example, Harold Cherniss, "Plato as Mathematician", *Review of Metaphysics* 4, no. 3 (1951): 418–419. The role of Plato as a mathematician is problematic too: "the main problem in discussing Plato as a mathematician… is that most of the statements dealing with mathematics are, to the modern reader, at least couched in vague language", Roger Herz-Fischler, *A mathematical history of division in extreme and mean ratio* (Waterloo: Wilfrid Laurier Univ. Press, 1987), 79.

3 Even if Plato, in most of his dialogues, is not explicit concerning the form feature of numbers, and he does not use forms as the most economic explanation for the "existence" of numbers, he is usually charged with mathematical realism by scholars and mathematicians. It is Aristotle, who, in his struggle to reject Plato's assumptions on numbers (or what he thought that Plato assumed about numbers), states that for Plato numbers are forms, criticizing Plato for having separated numbers from things. Aristotle's

rejection of Plato's philosophy of mathematics is part of his overall refusal to accept any kind of theory of forms, coming thus as a natural objection to Plato's conception of numbers. At the same time, Aristotle extrapolates many features of the forms to numbers, assuming that Plato thought in the same manner concerning numbers. As in the case of forms, Aristotle argues that numbers should not be separated from things, but rather that they are the product of counting things: one cannot have numbers without things which are counted.

4 P. Shorey, *The Unity of Plato's Thought* (Chicago: University of Chicago Press, 1968).

5 H. Cherniss, *The Riddle of the Early Academy* (Berkeley: University of California Press, 1945).

6 P. Pritchard, *Plato's Philosophy of Mathematics* (Sankt Augustin: Academia Verlag, 1995).

7 W. Tait, "Noesis: Plato on exact science", *Reading Natural Philosophy: Essays to Honor Howard Stein*, ed. D. Malament, (Illinois: Open Court), 11–30.

8 A. Wedberg, *Plato's Philosophy of Mathematics* (Westport, CT: Greenwood Press, 1977). One of the earliest defense is that of J. Adam, *The* Repu*blic of Plato,* (Cambridge: Cambridge University Press, 1902). For a critique of Adam, see P. Shorey which argues for number-form theory, "Ideas and Numbers Again," *Classical Philology* 22, no. 2 (April 1, 1927): 213–218.

9 "For a Platonist the Forms are yet more real and still more fundamental to explaining the scheme of things than the objects of mathematics" M. Burnyeat, "Plato on why mathematics is good for the soul", in T. Smiley, ed., *Mathematics and Necessity* (Oxford: Oxford University Press, 2000), 1–82. See also M. Burnyeat, "Platonism and Mathematics: A Prelude to Discussion", in *Mathematics and Metaphysics in Aristotle,* ed. A. Graeser (Bern & Stuttgart: Haupt, 1987), 212-40. Burnyeat thinks that "none of those who are sceptical of Aristotle's repeated and unambiguous ascription to Plato of a doctrine of intermediates has ever told us how mathematics *could* be about Forms instead". (Burnyeat, *Op.Cit.*, 229).

10 It is not clear if there is an exact correspondent in Greek for "pair". Perhaps it should be taken in the following manner: (a,b), (b,c), (a,c).

11 Couple.

12 It is customarily assumed that the partner of the dialogue with Parmenides is not Aristotle, the philosopher, but one of the Thirty Tyrants who ruled Athens after it was defeated by Sparta, in the Peloponnesian War.

13 It is worth noticing that for Aristotle, "the infinite by addition is the same thing as the infinite by division" (*Physics*, 206b3). Accordingly, for Aristotle there should be no significant distinction between progressive infinite (a) and the one by division (b). But is it the same for Plato? In the chapters 4-8 of book 3 of the *Physics*, dedicated to the problems of the infinite, Aristotle mentions

several times how Plato conceives infinity: "the Pythagoreans identify the infinite with the even… But Plato has two infinites, the Great and the Small" (203a10-15). Further, Aristotle takes that Plato "made the infinites two in number, because it is supposed to be possible to exceed all limits and to proceed *ad infinitum* in the direction both of increase and of reduction. Yet though he makes the infinites two, he does not use them. For in the numbers is not present the infinite in the direction of reduction, as the monad is the smallest; nor is the infinite in the direction of increase, for he makes numbers only up to the decad" (Aristotle, *Physics*, 206b30-33). In the *Parmenides* argument it seems that Plato used at least the infinite in the direction of increase, since there is no ending for numbers. It is not clear if "the Small" infinite (203a15) must stop at unity or, as I would like to stress is the other part of the one, its division. Unity is in both directions many. One could take into account an important difference between modern and ancient understanding of mathematical infinity. Compared with modern mathematics, which takes the infinites as gravitating around zero, {..., -4, -3, -2, -1,   0,   1, 2, 3, 4, ...}, for Greek mathematics and especially for Plato, the two infinities could be something like that (if one accepts that it is possible to go beyond unity, and if we take Zeno's paradox of motion as a problem concerning the division of unity): … 1/4, 1/3, 1/2,   1,   2, 3, 4, ….

14 (Ross' translation). τὸ μὲν οὖν τὸ ἓν καὶ τοὺς [30]ἀριθμοὺς παρὰ τὰ πράγματα ποιῆσαι, καὶ μὴ ὥσπερ οἱ [31] Πυθαγόρειοι, καὶ ἡ τῶν εἰδῶν εἰσαγωγὴ διὰ τὴν ἐν τοῖς λό-[32]γοις ἐγένετο σκέψιν (οἱ γὰρ πρότεροι διαλεκτικῆς οὐ μετεῖ-[33]χον), τὸ δὲ δυάδα ποιῆσαι τὴν ἑτέραν φύσιν διὰ τὸ τοὺς [34] ἀριθμοὺς ἔξω τῶν πρώτων εὐφυῶς ἐξ αὐτῆς γεννᾶσθαι ὥσ-[988a1]περ ἔκ τινος ἐκμαγείου. Primavesi's new revised edition of the Greek text does not differ on this passage from that of Ross. See Carlos Steel ed., *Aristotle's Metaphysics Alpha: Symposium Aristotelicum* (Oxford: Oxford University Press, 2012).

15 For some scholars this association is artificial. See, for example, Harold Fredrik Cherniss, *Aristotle's Criticism of Plato and the Academy* (Johns Hopkins University Press, 1944), 194-195. For a critique of Cherniss see Steel, *Plato as seen by Aristotle* in Steel ed., *Aristotle's Metaphysics Alpha*, 189-190.

16 In general it is thought that Aristotle does not refer at all to the *Parmenides*. For a discussion of the pro and cons arguments see: Donald J. Allan, "Aristotle and the *Parmenides*" in Ingemar Düring and Gwilym Ellis Lane Owen, *Aristotle and Plato in the Mid-Fourth Century: Papers of the Symposium Aristotelicum Held at Oxford in August, 1957* (Elanders Boktryckeri Aktiebolag, 1960), 133-144. Also A. E. Taylor, *The Parmenides of Plato* (Oxford: Clarendon Press, 1934), especially Appendix C: Aristotle and the *Parmenides*, 128-134.

17 David Amirthanayagam, "Plato and the Measure of the Incommensurable. Part II. The Mathematical Meaning of the Indeterminate Dyad" in *The St.*

John's Review Volume XLVI (2002), 25-62, 43. This is one of the common positions of modern scholarship, namely that Plato never thought about numbers as being generated.

18 W. D. Ross, *Aristotle's Metaphysics* (Oxford: Clarendon Press, 1924), 174.

19 W. D. Ross, *Volume VIII Metaphysica, The Works of Aristotle Translated into English* (Oxford: Clarendon Press, 1928), 987b, footnote 1.

20 W. D. Ross, *Aristotle's Metaphysics*, 174. In *Plato's Theory of Ideas*, Ross thinks that "this proof is rather perfunctory, since it makes no provision for prime numbers other than 2 and 3". See William David Ross, *Plato's Theory of Ideas* (Clarendon Press, 1951), 187.

21 W. D. Ross, *Volume VIII Metaphysica*, Book A6, 987b, footnote 1.

22 This statement contrasts with that from *Met.* 1081a, 14, in which it is stated that: "*Number* comes from the 1 and the indefinite dyad, and the principles and the elements are said to be principles and elements of number". Scholars do not agree if this statement is ascribed by Aristotle to Plato or to some of his followers. Its meaning is problematic and addresse a complicated challenge for historians of philosophy. Instead of clarifying Plato's understanding of numbers, these type of statements create ambiguities: e.g. "but what this apparently simple statement means has remained a mystery until modern times", Ivor Bulmer-Thomas, "Plato's Theory of Number", *Classical Quarterly* 33, no. 02 (1983): 375.

23 Even if Ross rejects an appropriation between A6 and the *Parmenides*, he thinks that the indefinite dyad ascribed by Aristotle "might be assigned to 2… such as [it] is expressed in *Parmenides*." Ross, Aristotle's *Metaphysics, 175.*

24 Julia Annas, *Aristotle's Metaphysics: Books M and N* (Clarendon Press, 1988), 48. John Dillon is more cautious and argues that, later, with Speusippus an ontological reading of the *Parmenides* starts, which, in the case of the second hypothesis, implies "an account of how One, when combined with the indefinite Dyad (under the guise of 'Being') produces, first the whole set of natural numbers, and then progressively, the various lower levels of reality", John Dillon, Syrianus's *Exegesis of the Second Hypothesis of the Parmenides: The Architecture of the Intelligible Universe Revealed*, in John Douglas Turner, Kevin Corrigan (ed.), "Plato's Parmenides and Its Heritage: Volume II: Reception in Patristic, Gnostic, and Christian Neoplatonic Texts" (Society of Biblical Literature Writings from the Greco-Roman World Supplement), 133.

25 Alfred Edward Taylor, *Aristotle on His Predecessors. Being the First Book of His Metaphysics, Translated from the Text Edition of W. Christ, with Introduction and Notes by A.E. Taylor* (Chicago: Open Court Publishing Company, 1907), 104.

26 "Through confounding the truth and the reference of the statement 'one is' in Parmenidean fashion, Plato treats 'one' and 'is' as belonging to *the one*

that is, and so by an easy step takes *one* and *being* to be its parts." Malcolm Schofield, "A Neglected Regress Argument in the *Parmenides*", The Classical Quarterly, 23 (1973), 44.

27 Δύο refers to two things and not exactly to *number* two. In Greek mathematics, number (*arithmos*) refers not to abstract entities, but to numerosities. In the *Parmenides*, there is still an ambiguity if one should conceive δύο as a proper abstract number (closer to a more modern understanding of number) or, in a tradition established by Greek mathematics, to a set of units.

28 First of all, in a tradition established by Pythagoreans, Plato would think about the indefinite dyad that is a metaphysical principle, while *duo* is a set of two members which would correspond to number two. Also, the *one* of the *one is* (of the pair relation *one-is*) is not the number one.

29 Plato, *Parmenides* in *Complete Works*, ed. John Madison Cooper and D. S. Hutchinson, trans. Mary Louise Gill and Paul Ryan (Indianapolis: Hackett Publishing, 1997), 377. All references to Plato in this paper are quoted from the John M. Cooper *Plato: Complete Works*.

30 R. E. Allen thinks that in English the argument lacks the force that it has in Greek: "The exact force of his argument cannot be reproduced in English. Greek possesses, as English does not, a dual as well. as a singular and plural; when Parmenides argues that since it is possible to mention Unity and to mention Being, each of two has been mentioned, the English „two" is more explicit than the text, which contains only the genitive dual αὐτοῖν. It is from this feature in the syntax of his language that Parmenides goes on to infer that both have been mentioned, and that since both have been mentioned, two have been mentioned." Plato, *The Dialogues of Plato, Volume 4: Plato's Parmenides, Revised Edition*, tr. R. E. Allen, (New Haven: Yale University Press, 1998), 262. In a presumably genus-species reading, there are two possibilities: a) two (as a species) is derived from ἄμφω (the genera), b) ἄμφω (as a species) presupposes already *two* (as genera). In the case of a) it is taken for granted that cardinality is a derivation of *ampho*, while for b) the cardinality is already there before analyzing ἄμφω. Plato operates a distinction between them, and he grants priority to ἄμφω as opposed to the counted *two*. Even if the pair seems to be a species of the cardinality of two, in this instance Plato conveys a different conception (142d2): ὢ δ᾽ ἂν ἄμφω ὀρθῶς προσαγορεύησθον, ἆρα οἷόν τε ἄμφω μὲν αὐτὼ εἶναι, δύο δὲ μή; ("Can things that are correctly called 'both' be both, but not two?").

31 Ross in *Aristotle's Metaphysics*, Oxford, 196.

32 See Taylor, *Aristotle on His Predecessors*, 120.

33 One can note here that Plato takes now 3 as being 3x1, and not 2+1 as in 143d7.

34 Cornford thinks that "Plato evidently includes addition and starts with that when he adds one term to another to make two, and two to one to make three". F. M. Cornford, *Plato and Parmenides* (Routledge, 2000), 141.

35 See *Plato's Phaedo* edited with introduction and notes by John Burnet (Oxford: Clarendon Press, 1963), 103.

36 For Brumbaugh, since "three is odd, and two even" (143d8), "the proof moves on from logistic or set theory to arithmetic, the theory of numbers treated as classes". R. S. Brumbaugh, *Plato on the One: the Hypotheses in the* Parmenides (Yale University Press, 1961), 97. Nevertheless, Brumbaugh does not insist more on the issue, and he takes the "definition of "twice" and "thrice" as relations between defined numbers." (*Ibidem*).

37 Thomas Little Heath, *A History of Greek Mathematics* (Oxford, The Clarendon Press, 1921), 72.

38 It is most probably that this was an alien claim for his contemporaries, who must have perceived number as the product of addition, subtraction, dividing etc. However, it is an open question if Plato makes the distinction between the form *F-ness* and number *n*. For a short review see David Galoop's notes on Plato, *Phaedo* (Oxford: Clarendon Press, 2002), 200.

39 But what Plato implies is far from clear, just like, for some scholars, "it remains unclear whether the Form of Two, for instance, is a collection of two ideal units or whether it is simply Twoness", John Cleary, "Aristotle's Criticism of Plato's Theory of Form Numbers", in Gregor Damschen, *Platon und Aristoteles-sub ratione veritatis: Festschrift für Wolfgang Wieland zum 70. Geburtstag* (Vandenhoeck & Ruprecht, 2003), 6.

40 Allen thinks that "Parmenides' argument is silent on the question of whether numbers are pluralities of units, or whether they are Forms, or whether they are, perhaps, 'intermediates'". R. E. Allen, "The Generation of Numbers in Plato's Parmenides", in *Classical Philology* 65, (1970), 33.

41 W. K. C. Guthrie, *A History of Greek Philosophy: Volume 1, The Earlier Presocratics and the Pythagoreans* (Cambridge: Cambridge University Press, 1978), 240.

42 Gerald James Whitrow, *Time in History* (Oxford: Oxford University Press, 1988), 40.

43 Criticizing Pythagoreans, Aristotle (*Met.* N. 1091a22-23) thinks that "it is strange also to attribute generation to eternal things, or rather this is one of the things that are impossible". However, for Pythagoreans some numbers are generated, while others not. For Aristotle, the claim that Plato had the numbers generated from *Met.* A. 987b22-35 doesn't seem to contradict a previous statement at *Met.* A, 987b16 that for Plato "the objects of mathematics are eternal and unchangeable". Here two possibilities of interpretation can be formulated: a) either Aristotle did not realize the inconsistency, b) either Aristotle did not see the generation of numbers as opposite to that of the eternity of numbers, the generation being conceived as not being in time – therefore a technical description (but this would contradict his reading of the *Timaeus* – as a generation in time).

44 Annas, 211. According to Allen, in this case "numbers are simple essences incapable of analysis into ontologically prior and posterior elements". Plato's aim is not a *ratio essendi*, but a *ratio cognoscendi*. Allen, *Plato's* Parmenides, rev. ed., 266.

45 *Ibidem* 265.

46 Robert G. Turnbull, *The Parmenides and Plato's Late Philosophy: Translation of and Commentary on the Parmenides with Interpretative Chapters on the Timaeus, the Theaetetus, the Sophist, and the Philebus* (University of Toronto Press, 1998), 73.

47 One of the major questions concerning Plato's philosophy is whether one can organize the platonic dialogues into a unitary philosophical system built around the theory of forms. Contemporary scholarship on Plato is divided in two apparently irreconcilable positions. On the one hand, there are scholars (Ryle, Robinson, Owen, McDowell, etc.) who argue that one cannot systematize Plato around the theory of forms (1) since consistent references to the theory forms are missing in late dialogues, thus Plato didn't endorse his theory after the *Parmenides*. On the other hand, there are scholars (Cornford, Ross, Sedley, Chappell, etc.) who think that (2) even if the theory of forms is not pointed out in the late dialogues, the reader should always bear it in mind as the underlying reference system, since Plato did maintain a unitary philosophy throughout all his dialogues. According to the *Revisionist* position (1) Plato consistently revised his philosophical commitments and one cannot resort to the theory of forms as explanatory for late dialogues, whereas with the *Unitarian* view (2), a consistent effort of harmonizing late dialogues with early dialogues is needed, since even if there is no explicit reference to the theory of forms (with the exception of *Timaeus*), it remains the principal structure of platonic thinking.

48 In the *Sophist* 255e, Plato gives to *difference* the same ontological power as in the *Parmenides* 143a-b.

49 William David Ross, *Plato's Theory of Ideas* (Oxford: Clarendon Press, 1951), 100.

50 Rejecting the idea that the subject matter of the *Parmenides* could refer to the Parmenidean monism, Runciman argues that "although certain arguments of the second part could be construed as referring to Parmenidean monism, it is clearly impossible so to interpret them all; and if Plato wished to discuss Parmenidean monism, he would not have done it in this intermittent way". Thus "But the ambiguities of the second part do not invalidate the contention that it is nevertheless the form of unity which is under discussion throughout." Walter Garrison Runciman, "Plato's Parmenides", *Harvard Studies in Classical Philology 64* (1959): 101.

51 The presupposition that of course for which "there is no internal evidence whatsoever" that it is discussed Parmenides' Monistic theory. Gilbert Ryle, "Plato's 'Parmenides'", *Mind*, New Series, 48, no. 190 (April 1, 1939): 143.

52 “The quotations from the poem and the references to it are so frequent in Plato’s writings that we may be sure when Plato was writing the Parmenides he had nothing more vividly before his mind than the poem which he mentions whenever he talks about the paradoxes of being.” Harold Cherniss, “Parmenides and the *Parmenides* of Plato”, *American Journal of Philology* (1932): 130.

53 Plato “wanted to clear up the relationship between his own doctrine and the Eleatic thesis of One Being”. W. K. C. Guthrie, *A History of Greek Philosophy: Volume 5, The Later Plato and the Academy* (Cambridge University Press, 1986), 57. Plato “wanted to clear up the relationship between his own doctrine and the Eleatic thesis of One Being”.

54 Turnbull, *The Parmenides and Plato’s Late Philosophy*.

55 Scolnicov argues that: “As the Parmenides will make clear, Parmenidean ascription of being is ‘transparent’. As Plato shows in Argument I, nothing is added to the Parmenidean one when it is said to be. To say ‘the one’ and to say ‘the one is’ is to say the same thing.” See Samuel Scolicov, *Plato’s Parmenides* (University of California Press, 2003), 18. Or, “Argument II is, together with the related Arguments III, V, and VII, an explication of *μέθεξις*, as opposed to Parmenidean being”, *Ibid.* 96.

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