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THE SYMBOLIC FUNCTION OF MATHEMATICS IN ERNST CASSIRER'S PHILOSOPHY OF CULTURE

Abstract

Cassirer's philosophy of symbolic forms offered the theoretical framework for a unified study of culture, including such nonrational forms assumed by human understanding of the world as mythical thinking. At the same time, Cassirer defended the role of mathematics and natural science as models of rationality in the Kantian sense. This paper offers a discussion of the role of mathematics in Cassirer's philosophical project, given the fact that he first developed the notion of symbolic form in order to account for the rationality of theory change in physics. The historical perspective of Cassirer's approach suggests that rationality depends not so much on the assumptions of some specific theory as on the unifying power of mathematical method. He argued for a model of rational thinking which, owing to its symbolic character, can be articulated in various ways without being contradicted by the fact that there are different symbolic forms.

Keywords: Cassirer, mathematical method, neo-Kantianism, symbolic form.

From Neo-Kantianism to the Philosophy of Symbolic Forms: The Problems of a Unified Theory of Culture

Ernst Cassirer was born in 1874 in the German city of Breslau (now Wrocław, Poland). During his studies in philosophy at the University of Berlin, he took a course on Kant taught by Georg Simmel. In particular, Simmel recommended to his students *Kants Theorie der Erfahrung* (1871) by the founder of the Marburg School of neo-Kantianism, Hermann Cohen. After reading Cohen's book, Cassirer decided to move to Marburg to complete his education under the supervision of Cohen and Paul Natorp. Cassirer studied at the University of Marburg from 1896 until

1899. He moved back to Berlin in 1903 and received his habilitation at the University of Berlin in 1906. Cassirer's early works, *Leibniz' System in seinen wissenschaftlichen Grundlagen* (1902), the first two volumes of *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit* (1906-1907), and *Substanzbegriff und Funktionsbegriff: Untersuchungen über die Grundfragen der Erkenntniskritik* (1910), were deeply influenced by Cohen's interpretation of Kant's transcendental philosophy.[1] Cohen pointed out that the method of transcendental philosophy differed from that of psychology both in its object and in its method. The object of transcendental philosophy is not so much actual experience as the a priori principles of knowledge, namely those principles that are independent of experience because they lie at the foundation of a possible experience in general. In the *Kritik der reinen Vernunft*, Kant (1787, 25) called "transcendental" all cognition that is occupied not so much with objects but with our manner of cognition of objects insofar as this is to be possible a priori. The transcendental analysis of the conditions of knowledge presupposes the distinction between sensations and spatiotemporal phenomena. Kant maintained that phenomena are given in the pure intuition of space and time in general. Experience is made possible by the application of the pure concepts of the understanding to the manifold of intuition.

On Cohen's view, experience in Kant's sense differed from psychological experience because of the general validity of a priori cognition. Therefore, Cohen distanced himself from Kant's theory of the faculties of the mind and identified experience with scientific knowledge. In order to clarify this point, in the introduction to his second book on Kant, *Kants Begründung der Ethik* (1877), Cohen characterized the transcendental method as follows:

> Experience being given, the goal of the transcendental inquiry is to find out the conditions of the possibility of experience. Insofar as these conditions make experience possible in such a way that this can be considered to be valid a priori (i.e., strictly necessary and generally valid), the same conditions form the characteristics of the concept of experience, and it is from the latter concept that all which has the epistemic value of objective reality has to be deduced. This is all the transcendental philosophy has to do. Experience is hence given in mathematics and in the pure part of natural science. (Cohen 1877, 24-25)[2]

Cohen referred to the method adopted by Kant in the *Prolegomena zu einer jeden künftigen Metaphysik die als Wissenschaft wird auftreten können* (1783). Given the fact that generally valid judgments are found in mathematics and physics, the conceptual analysis of their presuppositions provides us with a priori principles of knowledge in the above sense. It follows from Cohen's interpretation that the defining characteristics of a priori notions (i.e., necessity and generality) depend on their being implicit in scientific knowledge. In other words, scientific knowledge tends to assume a foundational role for the transcendental inquiry into the conditions of knowledge.

Cohen's characterization of the transcendental method motivated Cassirer to face the problem whether Kant's conditions were compatible with later scientific developments, such as non-Euclidean geometries, the mathematization of logic, relativistic physics, and quantum mechanics. In his 1921 book *Zur Einstein'schen Relativitätstheorie: Erkenntnistheoretische Betrachtungen*, Cassirer formulated the problem concerning a renewal of Kant's transcendental philosophy as follows:

> Kant believed that he possessed in Newton's fundamental work, in the *Philosophiae Naturalis Principia Mathematica*, a fixed code for physical "truth" and believed that he could definitively ground philosophical knowledge on the "factum" of mathematical natural science as he here found it; but the relation between philosophy and exact science has since changed fundamentally. Ever more clearly, ever more compellingly do we realize today that the Archimedean point on which Kant supported himself and from which he undertook to raise the whole system of knowledge, as if by a lever, no longer offers an unconditionally fixed basis. (Cassirer 1921/1923, 352-53)

Are there a priori principles of knowledge? On Cassirer's view, a solution to this problem according to the transcendental method required a reformulation of the notion of a priori in terms of anticipation of possible experience. That which is presupposed a priori, in Cassirer's sense, is not so much a set of allegedly immutable truths as a range of hypotheses including all possible specifications to be found in experience. Mathematics plays an a priori role because it provides the appropriate tools for the hypothetico-deductive kind of reasoning that is characteristic of theoretical physics. Owing to the use of mathematical method, the objectivity of physical theories does not depend directly on empirical facts. As the French physicist and historian of science Pierre Duhem (1914, 298) put it, in order to provide the basis for the development of a physical theory, empirical facts have to be transformed and put into a "symbolic form." Cassirer (1921, 96, and note) borrowed this expression from Duhem to indicate the fact that the interpretation of measurements presupposes theoretical principles and in the latter general functions of coordination between the principles and the empirical manifold.

On account of the symbolic form of physical reality, Cassirer contrasted objectivity as an epistemic value with the idea of an absolute reality: the possibility of univocally establishing the meaning of the symbols correlated with physical events, and therefore the objectivity of the theory, depends not on direct reference, but on the generality of the frame of reference. At the same time, he believed that a philosophical account of reality required a broader perspective than that offered by the theory of knowledge. He wrote:

> It is the task of systematic philosophy, which extends far beyond the theory of knowledge, to free the idea of the world from this one-sidedness. It has to grasp the *whole system* of symbolic forms, the application of which produces for us the concept of an ordered reality, and by virtue of which subject and object, ego and world are separated and opposed to each other in definite form, and it must refer each individual in this totality to its fixed place. (Cassirer 1921/1923, 447)

It followed that symbolic forms cannot serve as the expression of "truth" and "reality" in their singularity but rather as a system, which ought to include the forms of the theoretical, ethical, aesthetic, and religious understanding of the world.

Cassirer developed his view in the *Philosophie der symbolischen Formen*, which appeared in three volumes in 1923, 1925, and 1929. In the introduction to the third volume, which is devoted to the phenomenology of knowledge, Cassirer made it clear that the implementation of his philosophical project presupposed a widening of his original Kantian and neo-Kantian perspective. He wrote:

> The Philosophy of Symbolic Forms is not concerned exclusively or even primarily with the purely scientific, exact conceiving of the world; it is concerned with all the forms assumed by man's understanding of the world. It seeks to apprehend these forms in their diversity, in their totality,

> and in the inner distinctiveness of their several expressions. And at every step it happens that the "understanding" of the world is no mere receiving, no repetition of a given structure of reality, but comprises a free activity of the spirit. (Cassirer 1929/1985, 13)

The continuity between the philosophy of symbolic forms and Cassirer's earlier studies in the philosophy of science lies, nonetheless, in the fact that the history of mathematics and of physics – in Cassirer's reconstruction – most clearly showed the spontaneity which is characteristic of any symbolic form. The problem of formulating universal criteria of physical objectivity had its counterpart in the more general problem of overcoming one-sidedness after the divide between *Naturwissenschaften* and *Geisteswissenschaften*. Cassirer's solution was Kantian in spirit because it was based on the idea of freedom as a presupposition for both theoretical and practical uses of reason. In order to account for a larger variety of cultural phenomena than those considered by Kant, Cassirer maintained that the free activity of the mind in shaping human experience originates from the more fundamental level of symbolic thinking. This way of thinking is not a prerogative of reason, as it is characteristic of such nonrational symbolic forms as myth as well.

Owing to Cassirer's approach, and to his background in a variety of disciplines, his philosophy of symbolic forms offered a promising basis for the development of a unitary, but not hierarchical, theory of culture. In a similar way, Cassirer played a mediating role between the emerging traditions in twentieth-century philosophy, namely the so-called "analytic" and "continental" traditions. He had fruitful exchanges with leading figures of logical positivism, such as Moritz Schlick, Hans Reichenbach, and Rudolf Carnap, whose work in logic and the philosophy of science were seminal for the development of analytic philosophy in the United States and in the English-speaking world. And he was the main opponent of Martin Heidegger during the celebrated "International University Course" held in Davos, Switzerland, in 1929, when Heidegger's existentialist version of Husserl's phenomenology was about to become dominant in Germany and continental Europe after the advent of Nazism.[3]

Cassirer's philosophy draws increasing attention in current attempts to reconstruct the history of these traditions. In particular, Cassirer's work has been rediscovered in the English-speaking philosophical community, after Michael Friedman (2000, 159) indicated Cassirer as the most suitable source of ideas for finally beginning a reconciliation between the analytic and the continental traditions. Massimo Ferrari, Thomas Ryckman, and Friedman himself, among others, emphasized the relevance of Cassirer's epistemology to ongoing discussions in the philosophy of science, with special focus on the philosophical aspects of general relativity. Structural realists such as Steven French, James Ladyman, and Angelo Cei referred to Cassirer (1936) in support of their own account of such physical objects as quantum particles in relation to the problem of individuality and identity. Meanwhile, the ongoing publication of Cassirer's Nachlass initiated by John Michael Krois cast new light on the significance of his exchanges with scientists such as Einstein and with the logical positivists.

However, Friedman raised compelling objections against both Cassirer's philosophy of science and the philosophy of symbolic forms. He attributed to Cassirer a formalistic account of knowledge. It followed that empirical knowledge cannot be clearly distinguished from pure mathematics, on the one hand, and from coherent but arbitrary systems of metaphysics or myth, on the other (Friedman 1999, 27). In the following, I discuss Friedman's objections concerning Cassirer's philosophy of science. I argue that a contextualization of Cassirer's studies in the history and philosophy of science might shed light on the role played by mathematics in his architectonic of knowledge, both in the narrower sense of scientific knowledge and in the context of his philosophy of culture.

Cassirer's Argument for Continuity across Theory Change

According to Friedman, Cassirer offered a possible solution to a problem he was confronted with especially in his interpretation of Einstein's general relativity: is there continuity across theory change? To put it in a later terminology created by Thomas Kuhn (1962), the shift from classical mechanics to relativistic physics is a classic example of "paradigm shift." As a consequence of such a shift, subsequent paradigms are incommensurable: the theoretical terms of the new theory have completely different physical referents from those of the previous one. Therefore, Kuhn maintained that "an apparently arbitrary element, compounded of personal and historical accident, is always a formative ingredient of the beliefs espoused by a given scientific community at a given time" (Kuhn 1962, 4). As the range of espoused beliefs increases considerably outside the domain of experimental sciences, Kuhn's consideration calls into question the possibility of knowledge altogether. How to escape the conclusion that all knowledge is local?

Friedman (2005) interprets Cassirer's philosophy of science as an attempt to show that continuity across theory chance can be accounted for in terms of mathematical structures. He refers to the fact that Cassirer was one of the first philosophers to recognize that Euclidean geometry can be included in a general system of hypotheses and considered as a limiting case of non-Euclidean geometries. Similarly, the mathematical structure of general relativity can be proved to contain that of Newtonian physics as an approximate special case. However, this consideration does not provide a solution to the main problem: discontinuous changes affect the physical interpretation of abstract mathematical structures. Therefore, Friedman suggests that Cassirer's argument should be completed by the relativized conception of a priori principles proposed by Hans Reichenbach in *Relativitätstheorie und Erkenntnis apriori* (1920). Reichenbach (1920, 46) distinguished between two meanings of the notion of a priori in Kant's philosophy. On the one hand, a priori principles are supposed to be valid for all time. On the other hand, they are constitutive of experience insofar as they provide nonempirical presupposition for the definition of empirical concepts. In this sense, a priori principles can be identified with the coordinating principles linking mathematical structures to empirical reality, and might be subject to revision as in the case of Einstein's general theory of relativity.

Friedman's point is that, even in such a case, there can be continuity with regard to the meta-scientific, philosophical level of conceptual transformations. For example, he mentions the fact that Einstein was involved in the nineteenth- and early twentieth-century debate about the consequences of non-Euclidean geometry for the Kantian theory of space.[4]

The above reading of Cassirer seems to presuppose a classification of mathematical structures from a formal-logical viewpoint. However, Cassirer's argument for continuity across theory change depends not so much on the use of formal logic as on his insights into the history of mathematics. First, Cassirer observed that abstract concepts had been developed in the nineteenth century for the solution of specific issues in mathematics. One of the goals of these developments, in the works of mathematicians such as Bernhard Riemann and Felix Klein, was to better understand the connection between different branches of mathematics, as well as that between mathematics and physics. The kind of continuity that emerged from Cassirer's interpretation of these works depends not so much on the supposition of some underlying mathematical structures as on his view of the mathematical method. Second, a formalistic account of knowledge would contradict Cassirer's commitment to Kant's transcendental philosophy. The role of mathematical reasoning in Cassirer's architectonic of knowledge is due to the fact that mathematical method reflects an important characteristic of the method of transcendental philosophy: ontological assumptions are disregarded, so that hypotheses regarding the objects of experience can be classified from the most general viewpoint. In order to clarify this point, the following paragraph reconsiders Cassirer's view from 1907 to 1921. My supposition is that Cassirer's approach towards the history of mathematics in his earlier works lends plausibility on his argument for continuity across theory change as formulated in 1921.

Kant and Nineteenth-Century Geometry

After János Bolyai and Nikolay Lobachevsky developed consistent systems of non-Euclidean geometry, in the 1820s, both scientists and philosophers addressed the question whether the Kantian conception of space as a priori intuition ought to be refurbished or even rejected. The possibility of considering a variety of geometrical hypotheses appeared to contradict Kant's assertion that Euclidean geometry is grounded in a priori intuition, and suggested the view that the problems concerning the form of space are a matter for empirical investigation. Related to these problems, in the concluding remarks of his celebrated inaugural lecture of 1854 "Über die Hypothesen, welche der Geometrie zu Grunde liegen," Bernhard Riemann wrote:

> The answer to these questions can only be got by starting from the conception of phenomena which has hitherto been justified by experience, and which Newton assumed as a foundation, and by making in this conception the successive changes required by facts which it cannot explain. Researches starting from general notions, like the investigation we have just made, can only be useful in preventing this work from being hampered by too narrow views, and progress in knowledge of the interdependence of things from being checked by traditional prejudices. (Riemann 1854/1996, 661)

Riemann's views about space and geometry – especially in the interpretation of the German physiologist and physicist Hermann von Helmholtz (1870) – caused a debate about the question whether the origin of geometrical axioms is empirical or a priori. The debate culminated in the solution provided by the French mathematicians and physicist Henri Poincaré (1902, 66-67): geometrical axioms are neither a priori synthetic judgments in Kant's sense, in which case geometrical hypotheses could not be varied, neither empirical judgments to be discovered on a case by case basis, because the use of geometry in the description of empirical facts already presupposes that geometrical axioms have been stipulated.

Cassirer was one of the first philosophers to observe that Poincaré's solution was in line with the development of the mathematical method from nineteenth-century projective geometry to David Hilbert's "Grundlagen der Geometrie" (1899). In contrast to Euclidean definitions, which take concepts such as "point" and "straight line" as immediate data of intuition, these developments show that the properties of geometrical objects can be derived as the consequences of general rules of connection. This way of proceeding suggests that the object of mathematical investigation consists not so much of particular elements as of the relational structure as such. According to Cassirer, Hilbert's work is the clearest expression of this interpretation of the mathematical method, as it begins with a group of axioms, which we assume, and whose compatibility has to be proved (Cassirer 1910, 93).

Cassirer's (1907, 31-32) objection against Kant's assumption of a priori intuition as a source of mathematical certainty was that nineteenth-century mathematics deserved a purely logical derivation of the fundamental principles. As a priori intuition was Kant's middle term for the application of the concepts of the understanding to the empirical manifold, Cassirer's objection seems to undermine Kant's architectonic of knowledge altogether. Nevertheless, Cassirer argued for an equivalent architectonic of knowledge based on Kant's notion of synthesis in general. Kant distinguished the manifold of representations from the combination of a manifold or synthesis in general as follows:

> The manifold of representations can be given in an intuition that is merely sensible, i.e., nothing but receptivity, and the form of this intuition can lie a priori in our faculty of representation without being anything other than the way in which the subject is affected. Yet the *combination* (*conjunctio*) of a manifold in general can never come to us through the senses, and therefore

> cannot already be contained in the pure form of sensible intuition; for it is an act of the spontaneity of the power of representation, and, since one must call the latter understanding, in distinction from sensibility, all combination, whether we are conscious of it or not, whether it is a combination of the manifold of intuition or of several concepts, and in the first case either of sensible or non-sensible intuition, is an action of the understanding, which we would designate with the general title *synthesis* in order at the same time to draw attention to the fact that we can represent nothing as combined in the object without having previously combined it ourselves, and that among all representations combination is the only one that is not given through objects but can be executed only by the subject itself, since it is an act of its self-activity. (Kant 1787, 129-130)

For Kant, knowledge presupposes both the receptivity of sensibility and the spontaneity of understanding. At the same time, he made it clear that an act of the understanding is required in order for any combination in the object to be conceived. On Cassirer's view, Kant's clarification suggests that the receptive aspect of knowledge depends on its spontaneity in the formulation of hypotheses. The claim that "we can represent nothing as combined in the object without having previously combined it ourselves" appeared to be confirmed by the hypothetical status of geometrical assumptions in axiomatic systems.[5]

At the same time, Cassirer pointed out that the study of mathematical structures from the standpoint of the transcendental philosophy differs from formal logic because it is occupied not so much with the consistency of the hypothetico-deductive systems of mathematics as such but rather with the relationship between mathematical and empirical concepts. The goal of the transcendental philosophy is to prove that the same syntheses that lie at the foundation of mathematics rule over the cognition of the objects of experience as well (Cassirer 1907, 45).

Cassirer developed his view in his first major work in epistemology, *Substanzbegriff und Funktionsbegriff: Untersuchungen über die Grundfragen der Erkenntniskritik* (1910). On Cassirer's view, there is a tendency in the history of mathematics and of natural sciences to shift from concepts of substance to concepts of function. Both kinds of concepts are universal. The difference is that concepts of substance only admit relations of genus and species. Their formation, therefore, ultimately presupposes the existence of some individuals. On the contrary, a mathematical function represents a universal law, which, by virtue of the successive values which the variable can assume, contains within itself all the particular cases for which it holds. For example, Cassirer mentions Richard Dedekind's (1888) definition of natural numbers as "free creations of the human mind" based on the general notions of set theory and on the one-to-one correlation of each number to its successor in the series.

Cassirer's goal was to use the logic of the mathematical concept of function for the purposes of transcendental philosophy. Therefore, he emphasized that the field of application of this kind of logic is not restricted to mathematics alone. "On the contrary, it extends over into the field of the knowledge of nature; for the concept of function constitutes the general schema and model according to which the modern concept of nature has been molded in its progressive historical development" (Cassirer 1910/1923, 21). Thereby, the mathematical concept of function assumed the role played by the concept of time in Kant's *Kritik der reinen Vernunft*: the form of time contains the general conditions under which alone the concepts of the understanding can be applied to any object. Kant called the formal conditions of the sensibility, to which the use of the concept of the understanding is restricted, the "schema" of this concept, and he called the procedure of understanding with these schemata "schematism" of the understanding (Kant 1787, 179). On Cassirer's view, the schematism of the understanding corresponds to the fact that the general schema provided by the mathematical concept of function can be extended from the formation of algebraic and numerical concepts to that of the concepts of geometry and of physics.

Cassirer's analysis of the relationship between algebra and geometry is found in the third chapter of his work on "The Concept of Space and Geometry". Cassirer (1901/1923, 80) observed that in projective geometry – which flourished in the nineteenth-century after the works by Lazare Carnot, Jean-Victor Poncelet, Jakob Steiner, and Christian von Staudt – the object of inquiry consists not so much of the properties of a given figure as of the network of correlations in which it stands with other allied structures. Since ancient geometry, projections had been known to alter properties such as distances and the measure of angles. Therefore, from the standpoint of projective geometry, figures that are distinct from each other in ordinary geometry (e.g., circles, ellipses, parabolas, and hyperbolas) are classified as the same kind of figures (i.e., the conics). This fact suggests that the study of the projective properties of figures can attain the same generality of algebraic methods in the classification of the related structures. At the same time, Cassirer believed that projective geometry shed light on the formation of spatial concepts.

Cassirer relied, in particular, on the early works by the German mathematician Felix Klein: "Über die sogenannte Nicht-Euklidische Geometrie" (1871) and "Vergleichende Betrachtungen über neuere geometrische Forschungen" (1872). The latter was a written text which was distributed during Klein's inaugural address as newly appointed Professor at the University of Erlangen and became known as the "Erlager Programm."[6] Klein proposed a synthesis between two different traditions in projective geometry. On the one hand, he was familiar with Arthur Cayley's analytical treatment of projective metric by means of the algebraic theory of invariants developed by the British mathematicians George Boole and James Joseph Sylvester. On the other hand, Klein was introduced by his friend, the Austrian mathematician Otto Stolz, to the purely descriptive foundation of projective geometry by Christian von Staudt. Klein (1921, 51-52) reported that, in the summer of 1871, after his exchanges with Stolz, he came to idea that there must be a connection between projective metric and non-Euclidean geometry.[7]

Klein used Cayley's metric to provide the first example of a classification of geometries by means of group theory. Klein showed that geometric properties obtain as invariants relative to specific groups of transformations (i.e., functions from a set to another set or to itself); where the defining conditions for transformations to form a group are that the sum of operations of the group always gives an operation of the group and that, for every operation of the group, there exists in the group an inverse operation. Projective geometry is independent of the different possible hypotheses regarding the existence and the number of parallel lines because the group formed by projective transformations is wider than the group of Euclidean transformations, and contains both Euclidean and non-Euclidean hypotheses as special cases.

In *Substanzbegriff und Funktionsbegriff*, Cassirer used Klein's classification to interpret the notion of the form of space in general as follows:

> In this connection, projective geometry has with justice been said to be the universal "a priori" science of space, which is to be placed besides arithmetic in deductive rigor and purity. Space is here deduced merely in its most general form as the "possibility of coexistence" in general, while no decision is made concerning its special axiomatic structure, in particular concerning the validity of the axiom of parallels. Rather it can be shown that by the addition of special completing conditions, the general projective

> determination, that is here evolved, can be successively related to the different theories of parallels and thus carried into the special "parabolic," "elliptic" or "hyperbolic" determinations. (Cassirer 1910/1923, 88)

Cassirer associated Kant's form of space with the Leibnizian notion of space as the "possibility of coexistence" in general. Thereby, geometry could retain the status of an a priori science grounded in the form of space. At the same time, the relational aspect of Leibniz's notion shed light on the fact that the form of space provides us not so much with a specific axiomatic structure as with such a general classification of hypotheses as Klein's.

Cassirer's goal was to prove that this way of considering the form of space was necessary for the use of geometry in physics. We discuss Cassirer's argument in the next section. In the concluding part of this section, it noteworthy that, for Cassirer, not only did Klein's characterization of geometric properties as relative invariants of groups of transformation offer a model for the formation of concepts in physics, but it could be compared to the way of proceeding of transcendental philosophy. Cassirer wrote:

> Since we never compare the system of hypotheses in itself with the naked facts in themselves, but always can only oppose one hypothetical system of principles to another more inclusive, more radical system, we need for this progressive comparison an ultimate constant standard of measurement of supreme principles of experience in general. Thought demands the identity of this logical standard of measurement amid all the change of what is measured. In this sense, the critical theory of experience would constitute the universal invariant theory of experience, and thus fulfill a requirement clearly urged by inductive procedure itself. The procedure of the "transcendental philosophy" can be directly compared at this point with that of geometry. Just as the geometrician selects for investigation those relations of a definite figure, which remain unchanged by certain transformations, so here the attempt is made to discover those universal elements of form, that persist through all change in the particular material content of experience. (Cassirer 1910/1923, 268-69)

The inductive aspect of Cassirer's approach lies in the fact that the results of the universal theory of experience depend on the history of science. Therefore, he maintained that even the principles of Newtonian mechanics need not be taken as absolutely unchanging dogmas; Cassirer rather regarded these principles as the temporarily simplest intellectual hypotheses, by which we establish the unity of experience (*ibid.*). This claim suggests that Cassirer's approach did imply a relativized conception of the a priori, which became explicit in Cassirer's book on Einstein's general relativity. The misunderstanding in later reconstructions of his argument for continuity across theory change depends on the broader scope of Cassirer's theory of experience: the a priori role of the principles relative to specific theories presupposes a comparison of hypotheses at the meta-scientific level of the formation of concepts. Cassirer's model for such a comparison, in 1910, was Klein's group-theoretical treatment of geometry. In a similar way, Klein (1910, 21) maintained that projective geometry provided a rational ground for the assumptions of relativistic physics. He characterized special relativity as the theory of invariants of Minkovski's four-dimensional spacetime relative to the Lorentz group of transformations.

In 1921, Cassirer argument for continuity across theory change depended on the role of Riemannian geometry in Einstein's general relativity. A thorough discussion of Cassirer's argument would require us to add more details about Cassirer's and others' philosophical interpretation of general relativity.[8] For my present purpose, in the following section I limit myself to a few remarks about the philosophical significance of Riemann's geometry.

Cassirer's Argument in 1921

One of the most striking aspects of general relativity lies in Einstein's use of Riemannian geometry and its developments in Gregorio Ricci-Curbastro's and Tullio Levi-Civita's absolute differential calculus from the 1890s. Whereas projective metric sufficed to characterize Newtonian physics and special relativity as the invariant theories of the Galileian and of the Lorentzian transformation groups, respectively, the formulation of Einstein's equations presupposed completely different geometrical hypotheses about the structure of the spacetime continuum, which in general relativity is a four-dimensional manifold of variable curvature.

Riemann's work on manifolds of variable curvature did not receive much attention in the debate on the epistemological relevance of non-Euclidean geometry, as they appeared to be purely mathematical speculations. Cassirer was one of the first philosophers to recognize that this assessment had to be reconsidered after general relativity:

> The real superiority of Euclidean geometry seems at first glance to consist in its concrete and intuitive determinateness in the face of which all "pseudo-geometries" fade into logical "possibilities." These possibilities exist only for thought, not for "being;" they seem analytic plays with concepts, which can be left unconsidered when we are concerned with experience and with "nature," with the synthetic unity of objective knowledge. […] [T]his view must undergo a peculiar and paradoxical reversal. (Cassirer 1921/1923, 442-43)

On the one hand, deviations from the Pythagorean metric required a physical explanation: the value of the constant, which expresses the deviation, depends on the gravitational field, and can be neglected at the infinitesimal level and in the other cases in which the same results obtain according to special relativity. On the other hand, there is a reversal in what appeared to be "abstract" and "concrete." Now, relatively complex expressions have a physical meaning and Euclidean geometry is considered a limiting case.

Owing to his relativized conception of the a priori, Cassirer revised his former argument as follows. He identified the a priori of space as the more general function of spatiality that is expressed in the linear element of spacetime and pointed out the empirical meaning of the particular value of curvature. Nevertheless, he emphasized continuity with former physical theories as for the use of geometric concepts as "methodical anticipations" of experience. Since Riemannian geometry found a surprising application in Einstein s theory of gravitation, "the possibility of such an application must be held open for all, even the most remote constructions of pure mathematics and especially of non-Euclidean geometry" (Cassirer 1921/1923, 443). On Cassirer's view, the appropriateness of the transcendental method received a surprising confirmation as well, because the foregoing argument for the synthetic character of mathematics enabled Cassirer to account for the role of mathematics, including non-Euclidean geometry, in the formulation of new hypotheses. As Cassirer put it:

> A doctrine, which originally grew up merely in the immanent progress of a pure mathematical speculation, in the ideal transformation of the hypotheses that lie at the basis of geometry, now serves directly as the form into which the laws of nature are poured. The same functions,

> that were previously established as expressing the metrical properties of non-Euclidean space, give the equations of the field of gravitation. (Cassirer 1921/ 1923, 440)

To turn back now to Friedman's objection, the above quotation shows that Cassirer did not require continuity of mathematical structures across theory change in this case. Cassirer's argument for continuity, after general relativity, was based on the role of geometric concepts as methodological anticipations of possible experience. This consideration suggests that, if Cassirer's philosophy of science is to contribute to a possible solution to the Kuhnian problematic, the kind of continuity he is mainly concerned with is at the meta-scientific level of the transformation of concepts. The argument is that, although the symbolic language of Euclidean geometry adopted so far did not suffice for the correlation between space, time, and matter, which subsists according to the general theory of relativity, the theory of manifold first articulated mathematically over the second half of the nineteenth century provided Einstein with the appropriate tools for the discovery of the spacetime structure of general relativity.

As the symbolic function of mathematics in Cassirer's philosophy of science is strictly related to his account of physical reality in structural terms, the clarification of this point lends plausibility to his approach to the problem of reality in general. The consideration of different, even nonrational symbolic forms does not undermine the objectivity of knowledge, insofar as it confirms the unifying power of symbolic thinking in the articulation of human experience. It is because of the symbolic character of mathematics that the system of experience in the sense of transcendental philosophy is always capable of further generalizations. At the same time, Cassirer's historical perspective on rationality led him to extend his consideration to symbolic thinking as a more fundamental level of experience than logical thinking. In order to highlight this point, the following section sketches the connection between Cassirer's account of mathematical method and his broader perspective on rationality from the standpoint of the philosophy of symbolic forms.

The Task of Rationality and the Legacy of Enlightenment

Cassirer dedicated himself to the philosophy of mathematics and to the philosophy of science until the end of his life. In 1936, he published an important work on the philosophical implications of quantum mechanics. An overall presentation of his epistemological views is found in the third volume of the *Philosophie der symbolischen Formen* (1929) and in the fourth volume of *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit,* which was written in 1940 and appeared posthumously in 1957 (1950 in English translation). At the same time, he deepened his interests in a variety of disciplines, including anthropology, psychology, and intellectual history. In 1919, he was appointed Professor at the University of Hamburg. In 1920, while he was still working on *Zur Einstein'schen Relativitätstheorie,* he paid his first visit to Aby Warburg's Kulturwissenschaftliche Bibliothek in Hamburg, where he was impressed by the variety of perspectives on human culture offered by the ethnographic and aesthetic studies contained in the library. This visit, along with Cassirer's engagement in the debate about the geometry of space, seems to have contributed to his conviction that a philosophical account of "spatiality" deserved a more comprehensive study of the formation of the concept of space in its mythical, theoretical, and aesthetic meaning.[9] But Cassirer's visit to the Warburg library seems to have played an important role especially in Cassirer's broadening of his original perspective on symbolism.[10]

Notwithstanding the importance of this development in Cassirer's thought, he constantly relied upon his earlier account of mathematical method. In particular, his writings from the period between the end of the 1920s and the beginning of the 1930s, shed light on Cassirer's views about the role of scientific thinking and rationality in intellectual history and its potential. In the preface to his 1932 book *Die Philosophie der Aufklärung,* Cassirer characterized his approach toward the Enlightenment as follows:

> The age which venerated reason and science as man's highest faculty cannot and must not be lost even for us. We must find a way not only to see that age in its own shape but to release again those original forces which brought forth and molded this shape. (Cassirer 1932/1951, XI-XII)

Cassirer's defense of the values of the Enlightenment was strictly related to the symbolic and therefore constructive aspect which for Cassirer is characteristic of knowledge. The counterpart of the spontaneity of knowledge, in practical philosophy, is the conviction that rationality is not so much a fact, which we can take for granted, as a task to fulfill under the present circumstances. In this sense, Cassirer's stance in favor of rationality had a political significance.

In 1928, Cassirer was one of the few intellectuals in Weimar Germany to give a public speech about the history and legacy of the idea of republic. The political significance of this speech, whose written version appeared in 1929, and of his work from that period, was also due to the fact that in 1929-30 he acted as Rector of the University of Hamburg and as the first Jew to hold such a position in Germany. After the advent of Nazism, in 1933, Cassirer emigrated. He spent two years lecturing at the University of Oxford and six years at the University of Göteborg, Sweden. In 1941, he moved to the United States. He taught at the Yale University from 1941 to 1944 and at the Columbia University in 1944-45. Over these years he published an introduction to the philosophy of symbolic form for the English-speaking public under the title *An Essay on Man* (1944) and *The Myth of the State* (1946), which was devoted to the raise of fascism. Cassirer died in New York in 1945.

Under similar circumstances, intellectuals such as Edmund Husserl, Theodor W. Adorno, and Max Horkheimer took an opposite stance towards scientific thinking and the legacy of Enlightenment. They considered the mathematical expression of natural laws as a reduction of nature to technical processes which have nothing to do with human life and freedom. Not only did this view imply that there is an unbridgeable gap between natural sciences and the humanities, but – especially in Adorno's and Horkheimer's *Dialektik der Aufklärung* (1944) – it established a direct connection between the scientific approach to nature and authoritarian thinking. Cassirer's defense of the critical potential of rationality relied on a completely different – and well-documented – account of mathematical and scientific method. In this regard, it may be helpful to contrast Cassirer's view on mathematics with Husserl's. Owing to his scientific education, Husserl had a special interest in the philosophy of mathematics at the beginning of his career and was one of the first philosophers to appreciate the philosophical significance of Riemann's theory of manifolds. However, in his later works, he sharply contrasted the formal methods of mathematics with the method of the transcendental idealism.[11] In *Die Krisis der Europäischen Wissenschaften und die transzendentale Phänomenologie* (1936), Husserl characterized mathematical science as follows:

> Mathematics and mathematical science, as a garb of ideas, or the garb of symbols of the symbolic mathematical theories, encompasses everything which, for scientists and the educated generally, represents the life-world, dresses it up as "objectively actual and true" nature. It is through the garb of ideas that we take for true being what is actually a method – a method which is designed for the purpose of progressively improving, in infinitum, through "scientific" predictions, those rough predictions which are the only ones originally possible within the sphere of what is actually experienced and experienceable in the life-world. It is because of the disguise of ideas that the true meaning of the method, the formulae, the "theories," remained unintelligible and, in the naïve formation of the method, was never understood. (Husserl 1954/1970, 51-52)

A thorough discussion of Husserl's view would require us to take into account the phenomenological approach to the complex of individual, social, perceptual, and practical experiences which for Husserl form the life-world. For our present purpose, it is noteworthy that Husserl contrasts the objectivity of scientific theories with what he supposes to be the true nature of things as given to us. Therefore, he agrees with Cassirer that the mathematical method on account of symbolism fully abstracts from the immediate or intuitive aspects of experience. However, mathematics in Husserl's sense cannot provide us with a unified theory of experience in general, because his emphasis lies on the negative side of "abstraction" as loss of content. Instead of direct reference, mathematics and mathematical theories can only provide us with predictions that can be compared to one another in terms of accuracy.

By contrast, Cassirer dealt with the problem of perception in continuity with his account of mathematical symbolism. In the third volume of the *Philosophie der symbolischen Formen*, Cassirer noticed that even actual experience in everyday life presupposes the capacity to establish connections which is characteristic of symbolic thinking. In order to account for the use of the same capacity at the basic level of perception, Cassirer maintained that experienced phenomena are characterized by a form of "symbolic pregnance," which he described as follows:

> Just as, mathematically speaking, directed and nondirected quantities cannot simply be added together, we cannot, in our phenomenology and critical theory of knowledge, speak of "matters" and "forms," "phenomena" and categorical "orders" being "combined" with one another. On the other hand, we not only can but must determine every particular in respect to

> such orders, if experience is to come into being as a theoretical structure. It is participation in this structure that gives to the phenomenon its objective reality and determinacy. The symbolic pregnance that it gains detracts in no way from its concrete abundance; but it does provide a guarantee that this abundance will not simply dissipate itself, but it will round itself into a stable, self-contained form. (Cassirer 1929/1957, 203-204)

On Cassirer's view, symbolic thinking is not opposed to empirical contents, because it is the condition for recognizing these contents as parts of experience, both in the sense of everyday life and in that of scientific knowledge. The development of knowledge depends not so much on the accumulation of particular facts as on the inseparable connection between observable and nonobservable facts within a system of hypotheses. In other words, symbolic pregnance has its counterpart in the anticipatory role of the mathematical method in its empirical use.

Cassirer's view posed the problem of explaining how such different symbolic forms as myth and science are related. On the one hand, in the Introduction to the third volume of the *Philosophie der symbolischen Formen*, he emphasized the autonomy of nonrational forms of thinking, which cannot be derived from rational explanations. Therefore, in *An Essay on Man*, he maintained that the notion of symbol offered a more promising clue to human nature than rationality, as symbolic pregnance manifests itself in all of the forms assumed by human understanding and organization of experience. On the other hand, there seems to be a hierarchy of symbolic forms, from the most primitive to the most complex forms of science and of art in Cassirer's articulation of the system. The problematic aspect of such an order is that this seems to imply that rationality predominates at later stages of civilization.

With regard to the liberating role attributed to culture by Cassirer, Skidelsky (2008) considers Cassirer the last exponent of the humanistic tradition of the past two centuries. In this interpretation, one of the leading ideas of this tradition was that the manifestation of the symbolic capacity in culture and civilization shows a progressive direction. By contrast, twentieth-century philosophy was forced to rethink its basic presuppositions: "It could no longer treat as given the fact of science and culture" (Skidelsky 2008, 126). However, it seems to me that the above assessment overlooks the fact that the freedom of reason for Cassirer, as well as for Kant, is the condition of any critical attitude toward historical facts.

Cassirer himself had to call into question the said aspect of his philosophy, when, in *The Myth of the State*, he was confronted with the problem of accounting for the emergence of the modern political myths of the Nazi regime.[12] Nevertheless, even in his later works, he defended the legacy of the Enlightenment as a source of critical thinking also in relation to the present and reaffirmed the paradigmatic role of the sciences.[13] Without proposing a solution of the dialectical tension in Cassirer's philosophy of symbolic forms, I limit myself to conclude that the mathematical method on Cassirer's view assumed and retained a paradigmatic role as symbolic thinking. This does not necessarily mean that the history of human culture should reflect some hierarchy of values; the clarification of the constructive aspect of symbolism in relation to the mathematical method rather left the system of experience open to the possibility of deliberately articulating the same capacity in various ways.
(Endnotes)

NOTES

1 On Cassirer's education and on his relationship to Cohen and Natorp, see Ferrari 1988.

2 My translation from the German original: "Die Erfahrung ist gegeben; es sind die Bedingungen zu entdecken, auf denen ihre Möglichkeit beruht. Sind die Bedingungen gefunden, welche die gegebene Erfahrung ermöglichen, in der Art ermöglichen, dass dieselbe als a priori giltig angesprochen, dass strenge Nothwendigkeit und unbeschränkte Allgemeinheit ihr zuerkannt werden kann, dann sind diese Bedingungen als die constituieren Merkmale des Begriffs der Erfahrung zu bezeichnen, und aus diesem Begriff ist sodann zu deducieren, was immer den Erkenntniswerth objectiver Realität beansprucht. Das ist das ganze Geschäft der Transcendental-Philosophie. Die Erfahrung ist also in Mathematik und reiner Naturwissenschaft gegeben."

3 On Cassirer and the Vienna Circle, see Cassirer (2011). On the Davos disputation between Cassirer and Heidegger, see Aubenque et al. (1992); Friedman (2000); Gordon (2010).

4 See Friedman 2002.

5 In the foregoing quote, Kant was referring to a priori cognition in general. Kant clearly bore in mind the example of geometrical objects, if one compares this notion of synthesis with the following quote from the Preface to the second edition of the *Kritik der reinen Vernunft*: "A new light broke upon the first person who demonstrated the isosceles triangle (whether he was called 'Thales' or had some other name). For he found that what he had to do was not to trace what he saw in this figure, or even trace its mere concept, and read off, as it were, from the properties of the figure; but rather that he had to produce the latter from what he himself thought into the object and presented (through construction) according to a priori concepts, and that in order to know something securely a priori he had to ascribe to the thing nothing except what followed necessarily from what he himself had put into it in accordance with its concepts" (Kant 1787, XII).

6 The Erlangen Program is often mistaken for Klein's inaugural address (see Rowe1983). Klein's comparative review of the existing directions of geometrical research circulated as a pamphlet when Klein gave his inaugural address and became known as Erlangen Program, arguably because, after the second edition of 1893, Klein himself (e.g., in Klein 1921, 411-14) presented it as a retrospective guideline for his research.

7 On the sources of Klein's *Erlanger Programm*, see Wussing 1969, 133ff; Birkhoff and Bennet 1988; Rowe 1992.

8 For a thorough historical reconstruction of the early philosophical interpretation (and misinterpretations) of general relativity, see Hentschel 1990. For a reconsideration of Cassirer's and others' idealist perspective on the philosophical significance of general relativity, see esp. Ryckman 2005.

9 In 1931, he articulated this view in "Mythischer, ästhetischer und theoretischer Raum".

10 In a letter to Warburg dated 28 November 1920, Fritz Saxl, the deputy director of the library and Cassirer's guide during his visit, reported the following: "I began in the second room with the bookcase 'Symbol,' since I thought that Cassirer would find this an easy way to get to the library's problem. He immediately stopped short in surprise and explained that this was the very problem that had preoccupied him a very long time and on which he was presently working. Only a small portion of the literature on the concept of the Symbol that we have collected was known to him, and its orientation to the visual (the making visible of Symbolism in gesture and art), not at all." On the significance of the orientation of the library for the development of Cassirer's understanding of symbolism, see also Krois 2011. The English translation of the foregoing quotation is borrowed from Krois (2011, 15) as well.

11 See esp. Husserl's *Formale und Transzendentale Logik: Versuch einer Kritik der logischen Vernunft* (1929). For a thorough comparison with Marburg neo-Kantianism on the relationship between formal and transcendental logic, see Kern 1964.

12 Cassirer wrote: "When we first heard of the political myths we found them so absurd and incongruous, so fantastic and ludicrous that we could hardly be prevailed upon to take them seriously. By now it has become clear to all of us that this was a great mistake. We should carefully study the origin, the structure, the methods, and the technique of the political myths. We should see the adversary face to face in order to know how to combat him" (Cassirer 1946, 296). Although this self-criticism does not necessarily imply a revision of the system of symbolic forms, it clearly suggests that intellectual history from the viewpoint of such a system should take into deeper consideration the material conditions for the emergence of the political myths (see Pettoello 2013).

13 See, e.g., the following quotation from *An Essay on Man*: "The work of all the great natural scientists – of Galileo and Newton, of Maxwell and Helmholtz, of Planck and Einstein – was not mere fact collecting; it was theoretical, and that means constructive, work. This spontaneity and productivity is the very center of all human activities. It is man's highest power and it designates at the same time the natural boundary of our human world. In language, in religion, in art, in science, man can do no more than to build up his own universe – a symbolic universe that enables him to understand and interpret, to articulate and organize, to synthetize and universalize his human experience" (Cassirer 1944, 278).

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