New Europe College Yearbook 2006-2007

EMILIAN CIOC
SORANA CORNEANU
SORIN COSTREIE
CĂLIN NICOLAE COTOI
CĂLIN GOINA
DAMIANA GABRIELA OȚOIU
ADRIAN PAPAHAGI
MAGDA RĂDUȚĂ

Editor: Irina Vainovski-Mihai

ISSN 1584-0298

NEW EUROPE COLLEGE
Str. Plantelor 21
023971 Bucharest
Romania
www.nec.ro
Tel. (+4) 021 327.00.35, Fax (+4) 021 327.07.74
E-mail: nec@nec.ro

SORIN COSTREIE

Born in 1971, in Bucharest

Ph.D. University of Bucharest (2007) and University of Western Ontario (due 2008)
Lecturer, Department of Philosophy, University of Bucharest
Visiting Lecturer, Department of Philosophy, University of Western Ontario, Canada

Fellowships: 2003-2005 OGS, 2000-2004 IGSS, 2000-2002 PSGS, 1999 World Bank, 1997-1998 DAAD, 1996 TEMPUS, 1993 DAAD

Attended international conferences and seminars in Canada, Italy, Spain, Germany, USA, Poland
Several articles on philosophy of language, logic and mathematics

LEIBNIZ AND CLARKE. PHYSICS, METAPHYSICS AND MATHEMATICS

If we want to understand the mutual relationship of Science and Christianity we must try to re-interpret both of them. Our time is inclined to take science too much as an absolute truth and Christianity too much as an historical relic. The question of their relationship then usually takes the form: are Science and Christianity reconcilable? This question, I feel, is out of date. Rather we ought to ask whether they need each other, whether they can exist without each other.

Carl von Weizsäcker

Space and time. Two central points of philosophical interests. Are they relative or are they absolute? This is one the main issues of the well-known Leibniz-Clarke controversy. In fact, Clarke, one of the ablest English theologians of the late seventeenth century, who become known as a fervent defender of the thesis that no article of the Christian faith is contrary to reason, is representing here Newton's position, who held an absolute theory of space and time. Leibniz, another great believer in the rational order of the world, is defending, on the other hand, a relational theory of space and time. Their controversy started in November 1715, when Leibniz wrote a letter to Caroline of Anspach, at that time princess of Wales, complaining about the decay of Christian faith in England. This decay of religion was seen by Leibniz as being caused by the revitalization of materialism fed by certain ideas of Locke and Newton. The Princess gave Leibniz's letter forward to Clarke, who answered back to Leibniz and so the controversy began. Their correspondence comprises five letters from each side, Clarke's last letter was left unanswered by the death of Leibniz in November 1716.

The general accepted opinion is that the central topic of their controversy is the nature of space and time. However, as Leibniz himself points it out in his third letter to Clarke the

> dispute consists in many other things. The question is whether God does not act in the most regular and most perfect manner; whether his machine is liable to disorder, which he is obliged to mend by extraordinary means; whether the will of God can act without reason; whether space is an absolute being; also concerning the nature of miracles; and many such things, which make a wide difference between us. (L III 16)

As it is clear from this passage, the whole discussion concerns ultimately the theological relation between God and its creation. Does God 'put' the world in a pre-existent absolute spatial-temporal framework, or does He 'create' this framework simultaneously with the creation of the universe? In other terms, do space and time (pre)exist independently of the existence of objects or, as Leibniz claims, they are just relational, namely order of co-existences or, respectively, order of succession of objects?[1]

The end of my inquiry is to try to clarify the relations between physics, metaphysics and mathematics in Leibniz's case. This is not an easy task and much ink of pertinent Leibniz scholars was spread for its clarification. Although my analysis will focus mainly on the relation between physics and metaphysics in Leibniz-Clarke correspondence, yet, the other two fields, theology and mathematics will not be neglected, as well as previous works of Leibniz or of his contemporaries, which will be also targets of my analysis. Basically, there are three[2] main alternatives:

1. Leibniz's metaphysics derives from his logic;[3]
2. Leibniz's metaphysics derives from his physics;[4]
3. Leibniz's metaphysics is not grounded in anything else and, on the contrary, offers a fundamental basis for shaping and understanding the other two domains.[5]

My position concerning these points of views is to defend the last, and thus to say that Leibniz metaphysical inquiry shapes the other domains and not the vice versa. But let us start presenting Leibniz's points concerning the mentioned controversy and thus discuss his stand concerning these issues.

Leibniz's rejection of Newton's theory of absolute space and time is based mainly on two of his great principles: the Principle of Sufficient

Reason (hereforth, PSR) and The Principle of the Identity of Indiscernibles (hereforth, PII). Both principles are explicitly stated in the correspondence with Clarke: PSR as "nothing happens without a reason why it should be so rather than otherwise" (L III 2) and PII as "There is no such thing as two individuals indiscernible from each other". (L IV 4). Their importance is also explicitly acknowledged by Leibniz: "Those great principles[6] of sufficient reason and of the identity of indiscernibles change the status of metaphysics. That science becomes real and demonstrative by means of these principles, whereas before it did generally consist in empty words" (L IV 5)

Concerning space and time, Leibniz and Clarke stay in direct opposition. For Leibniz space and time were conceived as mere relations among material bodies, being thus ideal items or *entia rationis*, like mathematical entities, whereas for Clarke they are properties of God and thus sharing the necessary and absolute existence of the divine being.

Their disagreement concerns two main components: theology and metaphysics.[7]

As Ezio Vailati has nicely summarized, Leibniz' theological line of argumentation runs as follows:

> If absolute space and time existed, then either there would be something absolute and eternal besides God, or they would have to pertain to God's nature, with the result that God would have parts. Either way, the outcome would be theologically unacceptable. Clarke in effect restated the position he had put forth in the Boyle Lectures, namely, that space and time are properties of God, which however do not introduce parts in the divine being because space is essentially indivisible. Leibniz, quite reasonably, retorted that inseparable parts are still parts.[8]

Concerning the metaphysical line of attack, I will not present it here in detail, since there are already very good accounts of this confrontation in the literature.[9] However, two points are worth mentioning here. Firstly, as Vailati remarks, Leibniz's "standards arguments against absolute space and time were based on his views of the *continuum*. The notion of absolute space as an infinite whole composed of parts, in contrast to a mere infinite aggregate (not a whole) like the world, is incoherent, presumably because it impinges on the basic theorem, which Leibniz in contrast to Galileo and others refused to relegate to the infinite, that the whole is bigger than part". I'll deal with this point later. Secondly, as I have already

said, in rejecting Clarke's view of absolute time and space, Leibniz uses extensively PSR and PII: For example in his third letter, he argue that if space and time existed, then, taking into account that they are homogeneous, they may comprise identical parts, violating thus PII and ultimately PSR, since God's choice of where and when to create would be without reason. But let us consider now the first point.

I. Leibniz on the *Infinite*

I.1. The Big Infinity or the infinite addition

It is well known that Leibniz is playing in the history of the infinite the role of the advocate of actual infinite, constructing a consistent alternative to a whole venerable tradition which comes from Aristotle, and where the main actor has been the infinite understood as potential. Maybe the most invoked passage supporting Leibniz's position in this sense is a portion from a letter to Foucher, where Leibniz says explicitly that:

> I am so in favour of the actual infinite that instead of admitting that Nature abhors it, as is commonly said, I hold that Nature makes frequent use of it everywhere, in order to show more effectively the perfections of its Author. Thus I believe that there is no part of matter which is not, I do not say divisible, but actually divided; and consequently the least particle ought to be considered as a world full of an infinity of different creatures. (G I 416)

So far, so good, but the situation becomes more complicated because of Leibniz's quite explicit rejection of infinite numbers. Here Leibniz is using Galileo's puzzle with regard to natural numbers taken as an infinite series. In fact, we need two series, one is the series of natural numbers (1, 2, 3,…) and, for example, the series of the squares of the natural numbers (1, 4, 9,…). It is clear that, since each natural number has an unique square, there is an one-one correspondence between naturals and their squares. But thinking of the series as infinite collections of numbers and therefore considering the latter series as sub-series of the initial one, we will obtain an interesting and puzzling situation, namely that the elements of an infinite collection can be put in one-one correspondence with one of its own proper sub-collection. Strictly mathematically speaking, everything is fine and this feature will constitute in fact the hallmark[10] of

the infinite collections for mathematicians like Richard Dedekind[11] or Georg Cantor,[12] but, for a philosopher like Leibniz, this leads to an unacceptable situation. The acceptance of infinite collections leads to the negation of a very intuitive basic law, which says that 'the whole is greater than its proper parts'. Thus, on this metaphysical basis, Leibniz is forced, for the sake of coherence, to reject the actual existence of infinite collections.

Leibniz will characterize such collections as not being real wholes, and so, since unity means existence for him, infinite collections do not have actual existence; they could be found in thought, but not in reality. An interesting consequence of this line of argumentation will be that 'the universe as a whole' is a mere idea, with no counterpart in the actual reality. The universe is not, in fact, a real whole, it doesn't have a real unity and thus lacks actual existence. The concept was further developed in detail by Immanuel Kant,[13] who did claim that the idea of taking the universe as a whole is contradictory and represents one of the antinomies of the ration.

The part-whole axiom leads not only to the refutation of the concept of infinite collection, but also to the very concept of infinite numbers. The argument goes as follows. If we assume the existence of an infinite collections (of things), that means that we can count them and the result will be an infinite number. But for Leibniz that means that this number will be the last or the greatest number in the counting series, and, obviously, it will be a natural number as well. Contradiction, because the series on natural numbers do not admit any such number as the last or the greatest number.[14]

This intrinsic contradiction in the heart of the concept of infinite numbers could be also seen as an independent proof for rejecting the idea of actual infinite collections. It depends only on the starting point of our analysis. If we will begin with infinite numbers, then we can construct a set of them that will lead us to an infinite collection. We have only to identify the magnitude of this set with its cardinal number (or the "number of numbers" for Leibniz), which will be the last number in the increasing series. Since the last number is an infinite number the series will be infinite and thus the set as the collection of the members of the series will be infinite as well. The other way around, when we start by considering an infinite collection and we'll arrive at considering infinite numbers, was presented earlier.

Summing up the discussion up to this point, we have the paradoxical situation that Leibniz accepts the actual infinite but rejects infinite numbers and collections. This claim is paradoxical because it is in contradiction to the common intuition that whenever we accept that there are actually infinitely many things, simply by counting them, we will obtain the existence of infinite numbers and/or infinite collections as well.[15] This will entitle some commentators to regard his system as incoherent (G. Brown,[16] S. Levey), whereas other scholars will still maintain that Leibniz endorses a coherent philosophical view (R. Arthur, J. Benardete); there are others who seems to interpret Leibniz as coherent but maintaining two different kinds of infinite, a mathematical and a metaphysical infinite (A.W. Moore). Another type of position is to interpret him as internally coherent, like a consistent axiomatic system, but to say that his axioms contains errors (B. Russell).[17] Even though the presentation of this controversy lies outside the scope of this paper, I will present succinctly Richard Arthur's way out from this puzzling situation; he is drawing an interesting distinction, which seems to carry an important insight. For Arthur, Leibniz is consistent in his claims concerning infinity, and should be understood as endorsing an infinite that is actual but *syncategorematic*.[18]

> Leibniz has distinguished a perfectly middle ground between Cantor's actual infinite and Aristotle's potential infinite. Claims that it is some sort of unsound halfway house between the two are in error: it is quite coherent to say that there are actually infinitely many things without accepting Cantor's theory of the transfinite.[19]

I endorse a similar position,[20] namely that Leibniz seems to have a coherent view of the infinite, and I think that is important to distinguish between his mathematical and metaphysical ideas, which, although they are intrinsically interconnected, constitute two different levels of approaching the world. Moreover, it seems that Leibniz was aware of these difficulties and he has already drawn himself the distinction between an actual infinite that can be seen either as syncategorematic or as categorematic.

> It is perfectly correct to say that there is an infinity of things, i.e. that there are always more of them than one can specify, but it is easy to demonstrate that there is no infinite number, nor any infinite line, nor other infinite quantity, if these are taken to be genuine wholes. The Scholastics were taking that

> view, or should have been doing so, when they allowed a "syncategorematic" infinite, as they called it, but not a "categorematic" one. The true infinite, strictly speaking, is only in the absolute, which precedes all composition and is not formed by the addition of parts. (G VII 468)

In fact Leibniz's conception of infinite is even more complicated, since he is also considering a third alternative, that of a 'hypercategorematic infinite':

> There is a syncategorematic infinite, or a passive power having parts, namely, the possibility of further progression in dividing, multiplying, substracting, and adding. There is also a hypercategorematic infinite, or a potestative infinite, an active power having parts, as it were, eminently, not formally or actually. This infinite is God himself. But there is no categorematic infinite, or one actually having infinite parts formally. (G II 314f)

But as Laurence Carling points out, "it is important to realize, of course, that Leibniz's denial of the categorematic infinite is not a denial of the actual infinite".[21] Therefore, we can speak of infinite Leinizian numbers only in the syncategorematical sense; whenever we will pick out a certain number, there will always be another bigger number, and this process goes ad infinitum. But is improper to say that Leibniz endorses the categorematic view of the infinite numbers, where, independently of any given number, we can say that there is a number that is bigger that any finite number. The latter view characterizes for example Cantor's position with regard to transfinite numbers where ? is defined as the least transfinite number.

But, if the Leibniz's position is now quite clear and distinct from Cantor's, the jeopardy relies upon the menace of collapsing with Aristotle's position. So, what's the difference between Leibniz and Aristotle with regard to infinite? If we are taking Aristotle as holding the view of an potential infinite that is in principle 'untraversable',[22] then is clear that even though Leibniz did not accept to posit actually a Cantorian stable and firm infinite in mathematical thought, he would not agree that the infinity cannot be reached in principle in the actual world. The world is infinite for Leibniz. But the danger still persists in the sense that one could say that Leibniz accepts actual infinite in metaphysics, but rejects it in mathematics. This is in fact the position of A. W. Moore:

> Certainly Leibniz left no room for doubt that he regarded space, time, and that which is in space and time as infinite both by addition and by division, and in some sense actually so. (…)
> Borrowing the term introduced by the medievals, he said that nature was infinite only in a syncategorematic sense. That is, given any finite part of nature, there was always more to come. This sounds much closer to Aristotle. (…)
> Describing a straight line as a circle with an infinite radius was (in certain contexts) a convenient facon de parler. In general, then, Leibniz's views concerning the mathematically infinite did contain an important element of Aristotelian restraint.[23]

Based of the syncategorematic-categorematic distinction one could respond that, for instance, when a series goes ad infinitum, for Leibniz means that we are dealing actually with a real infinity of things, even though we are not able to deal with the infinite as such. Here, very important seems to be the unboundness of such series. No last term could be find here, because whenever we will pick out any term, immediately we will obtain its successor in that series and thus the entire process is a never ending story. On the other hand, I agree with Arthur that the distinction concerns the actual infinite and marks firstly a commitment to the real existence of infinite, and only after that it is coming with some clarification with regard to the actual infinity of things. Mathematically speaking, such commitment endorses a dynamic view of the infinite, in contrast with the Cantorian static view, where the infinite is an absolute given.

I.2. The Small Infinity or the infinite division

A second interesting Leibnizian puzzle concerning infinity is the one with regard to the infinite divisibility of the matter. Let us recall that for Leibniz matter is not only 'potentially divisible but actually divided'. Moreover, and I will discuss this point in the second part of the paper in a more detailed manner, Leibniz endorses that the bodies as actual things are discrete objects and thus we come upon a paradoxical view where each finite portion of matter comprises infinitely many finite parts. Thus the division of material things could go ad infinitum, which certainly implies that there are no minimal or ultimate material elements of the matter. There are no substantial atoms in the world exactly how a continuum line it is not made by points.

The puzzle of infinite division of the matter is well crystallized by John Bernoulli in the following challenging question, which appears in his correspondence with Leibniz: "You admit that any portion of matter is already actually divided up into an infinite numbers of parts; and yet you deny that any of these parts can be infinitely small. How is this consistent?". Leibniz's response was:

> I do not think it follows from (the infinite division of matter) that there exists any infinitely small portion of matter. Still less do I admit that it follows that there is any absolutely minumum portion of matter … Let us suppose that in a line is 1/2, 1/4, 1/8, 1/16, 1/32, etc., are actually assigned, and that all the terms of this series actually exists. You infer from this that there also exists an infinitieth term. I, on the other hand, think that nothing follows from this other than that there actually exists any assignable finite fraction, however small you please. (GM III, 529)

Thus, in order to grasp Leibniz's thought, it is quite important to see how this metaphysical process of the infinite division of the matter has a mathematical model in the sum of elements of the convergent series of 1/2, 1/4, 1/8, 1/16, 1/32… The idea behind this series is that such a process (of infinite division) is infinite in the sense that the series is unbounded. There is no last or least term, just because, as in the case of natural numbers, given any certain number (fraction) in this series, we can find very quickly a smaller number (fraction), which will be in fact its successor in this convergent series. But here, as in the case of natural numbers, we can only speak about a syncategorematic actual infinite, and it is not the case to asses the existence of a categorematic actual infinite. The latter may imply the actual existence of infinitesimal (small) magnitudes, idea strongly rejected by Leibniz; for him infinitesimals were mere facon de parler.

With regard to this mathematical model, Samuel Levey provides us with a very interesting analysis, presenting us two alternative ways of interpreting the infinite division of matter. One is called the 'diminishing pennies' model,[24] whereas the other is the 'divided block' model.[25] Both models consist in two different ways of viewing ontologically the infinite division of matter, and combined with Leibniz's own model ('folding tunic') entitles the author to draw the conclusion that Leibniz seems to be contradictory in the sense that his model does not fit very well his own ontological considerations concerning the infinite division of the matter.[26]

Let us consider now Leibniz's own model, which runs as follows:

> The division of the continuum must not be considered to be like the division of sands into grains, but like that of a sheet of paper or tunic into folds. And so although there occur some folds smaller than others infinite in number, a body is never thereby dissolved into points or minima. On the contrary, every liquid (i.e., all matter) has some tenacity so that although it is torn into parts, not all the parts of the parts are so torn in their turn; instead at any time they merely take shape, and are transformed; and yet in this way there is no dissolution all the way down into points…. It is just as if we suppose a tunic to be scored with folds multiplied to infinity in such a way that there is no fold so small that it is not subdivided by a new fold… And the tunic cannot be said to be resolved all the way down into points; instead, although some folds are smaller than others to infinity, bodies are always extended and points never become parts, but always remain mere extrema.[27]

It is clear that here when Leibniz speaks about parts which are infinite in number, he has in mind the syncategorematic sense of the term infinite. He is entitle to treat matter in this sense because here he is dealing with material bodies which are finite, even though they can be infinitely divisible, or moreover, more correctly, they are actually infinitely divided in finite parts. A second point that may be stress out is that Leibniz thinks there are no ultimate material constituents of the substantial world, no material atoms or points. Each portion of matter, whatever small it is, has extension and is be further divided to infinite. Last but not least, Leibniz confuses us here by discussing matter in a way in which it seems that it is continuous, when, on the contrary, he is very clear in other places that substantial bodies qua aggregates are discrete.

II. Leibniz on *Miracles*

There are two kinds of miracles for Leibniz: ones are supernatural, and required the direct intervention of God, and the others are natural, in the sense in which they are performed by angels and the laws of nature are not broken. They are 'natural' exactly in the sense in which there is, in this case, no violation of the physical laws. These miracles are performed by angels, also created beings, which differ from us in the sense in which what we do could be also seen as miraculous for inferior being as dogs or cats:

> These angels or these substances act according to the ordinary laws of their nature, being combined with bodies more rarefied and more vigorous than those we have at our command. And such miracles are only so by comparison, and in relation to us; just as our works would be considered miraculous amongst animals if they were capable of remarking upon them. The changing of water into wine might be a miracle of this kind. But the Creation, the Incarnation and some other actions of God exceed all the power of creatures and are truly miracles, or indeed Mysteries. If, nevertheless, the changing of water into wine at Cana was a miracle of the highest kind, God would have thereby changed the whole course of the universe, because of the connection of bodies. (T 249)

So, we can also distinguish between miracles of the first rank and miracles of the second rank. In the first class there are only the miracles done as a direct intervention of God, namely acts like:[28] creation, conservation, annihilation and incarnation.[29] In the second class there are the miracles of the second rank, in which, due to the intervention of angels, and in concordance with the laws of nature, some things are changed in the world in a miraculous manner. They are miraculous because such actions surpass our power of understanding.

As R. Adams interesting observed: "Leibniz certainly thought that free actions 'cannot be foreseen by the reasoning of any created mind' because of the complexity of the order on which they depend."[30] At this point it is worth to mentioned that a miracle could be seen at least from two perspectives. There are *epistemological* miracles, in the sense that the laws of the actual world cannot be comprehended by any created mind, and *ontological* miracles, namely that the order of the world is truly violated by a supernatural power. In the first case, we regard things or facts or phenomena as miraculous, because we cannot rationally understand what is going on there, due to the limits of our mind. In the second case, the law as it is known (and could be known) by human beings is broken, but it is violated the law as it is. In the first case we have in fact a rejection of the existence of miracles, because as long as we can not explain all the physical processes of the actual world, that means that we do not know entirely the physics of the world, because otherwise we could explain every physical process. Further, we could claim that either is just a question of time and technology until we will discover all the natural laws, or we cannot acquire such knowledge in the virtue of our intrinsic limitation of our understanding capabilities. In one case we will have a progressive elimination of miracle seen as a

'still unexplained phenomenon', in the other we can say that there are not really miracles. They are not really, because, for instance, for monkeys, a computer is, no doubt, a miraculous outcome, but it is so only because they do not have (the possibility of obtaining) a good explanation of that fact in their world.

> I maintained that an operation of God by which he should mend the machine of the material world, tending in its nature, as this author pretends, to lose all its motion, would be a miracle. His answer was that it would not be a miraculous operation because it would be usual and most frequently happen. I replied that 'is not usualness or unusualness that makes an miracle properly so called, or a miracle of the highest sort, but its surpassing the powers of creatures,[31] and that this is the general opinion of divines and philosophers; and that therefore the author acknowledge at least that the thing he introduces and I disallow is, according to the received notion, a miracle of the highest sort, that is, one which surpasses all created powers, and that this is the very thing which all men endeavor to avoid in philosophy. (L V 107)

Surpassing the powers of creatures could mean on one hand the *ontological power of acting*,[32] obtaining miracles of the first rank, or on the other hand the *epistemological power of understanding*[33] some things, dealing in this case with miracles of the second rank. But it would be a misunderstanding of the distinction, whether some body will see the miracles of the first rank only as 'ontological', and the miracles of the second rank only as 'epistemological'. It is obvious that creation surpasses the power of human being in both senses, as making and understanding something. We could not, of course, create the world as it is, and only we could not understand entirely the act of creation and even the world. On the other hand, even that the miracles of the second rank do not infringe the natural laws, this does not mean that we are able to produce similar effects. We have a limited power of creation and understanding, in the sense in which our potential infinity of acts and thoughts, has an real and actual limit, which cannot be surpassed by any human being.

It would be also worth mentioning that Leibniz defense of the existence some actions of God as miraculous, is in fact another way to avoid and reject Occasionalism and Deism. In the first case, God would be the only agent in the world, so, every acts of him will be miraculous, and consequently, will be no miracles in the world, as long as natural interventions are dismissed. In the second case, God's does nothing, so

neither miracles, so, again the conclusion that there are no miracles in the world.

So, talking about miracles, in Leibniz's account, 'true miracles' are only the supernatural actions of God, which go beyond the physics of this world,[34] and this is in fact in concordance with his doctrine of pre-established harmony, where within the physical world everything was fixed from the very beginning. This is an important thing, because otherwise we could not understand his metaphysical optimism, namely that we are living in the best possible world. It is the best universe, not because it does not comprise the evil, or at least a minimal amount of it, but because is the most complete world, with the most simple laws. The principle of plenitude, which consist in this maximal ratio between the simplicity of the rules that governs our world, and the richness of the phenomena of this actual world, could be understand entirely only from God's view. So, in order to conceive and understand Leibniz's world, we have to approach it from God's perspective. For example Carl von Weizsäcker[35] in his book *The World View of Physics*, acknowledges this important point about Leibniz's system:

> Probably the philosopher in Leibniz' sense has a similar experience when he surveys the world as a whole. It is no longer the mass of particular factual evil which is the object of his consciousness, but the meaning of the whole. If he succeed in seeing not with the eye of his limited human existence, but with the eye of God, then the choice between optimism and pessimism would be resolved.

III. Still Leibniz on *Miracles*. Physics and Metaphysics

Let us consider Leibniz's rejection of Newton's universal gravitation as miraculous in the sense that this would require the constant intervention of God into physical world, a thing which is seen by Leibniz as being contrary to the divine perfection of the creator who acts as a designer which sets up such a world that does not required any further intervention and adjustment. In a letter to Lady Masham, Leibniz says that:

> [I]t is no more comprehensible to say that a body acts at a distance with no means or intermediary than it is to say that substances as different as the soul and the body operate on each other immediately; for there is a greater gap between their two natures than between any two places. So the

> communication between these two so heterogeneous substances can only be brought about by a miracle, as can the immediate communication between two distant bodies; and to try to attribute it to I know not that what influence of the on the other is to disguise the miracle with meaningless words. (G III 354)

But this raises the following question: why did Leibniz ended up rejecting only universal gravitation, i.e. action-at-a-distance, and still accepting mind-body connection? For exemple, Gregory Brown's response to this puzzling question is that:

> Leibniz employed a strategy that he often employed in dispensing with apparent miracles, namely, that of offering what was, as he told Queen Sophie Charlotte, *only une fois pour toute* – "only a once-for-all explanation" (G III 347). Since the one, truly unavoidable miracle was that of creation, why not, leibniz seems to have reasoned, simply roll as many apparent miracles as possible into that initial, mega-miracle of creation. (…) *An une-fois-pour-toutes* arrangement at the moment of creation made it unnecessary for God to intervene miraculously in the course of nature.[36]

For instance, Newton (Query 23 of the 1706 edition of the *Opticks* / Query 31 of the 1717 edition) says explicitly that "God is a powerful ever-living agent, who being in all places, is more able by his will to move the bodies within his boundless uniform *sensorium*, and thereby to form and reform the parts of the universe, than we are by our will to move the parts of our bodies" (LC 181). That make Leibniz react, attacking the idea of (absolute) space as *sensorium Dei*, along with the idea that thus God is continuously (and miraculously!) intervening in the world. As a response to this, in his fourth letter, Clarke withdraws and reacts in the following way:

> That one body should attract another without any intermediate means, is indeed not a miracle, but a contradiction; for 'tis supposing something to act where it is not. But the means by which two bodies attract each other, may be invisible and intangible, and of a different nature from mechanism; and yet, acting regularly and constantly, may well be called natural… (G VII 388 / LC 53)

So, just because it is constant, for Clarke, God's intervention is natural and thus not miraculous, whereas for Leibniz, exactly because it is God's

intervention, it would be still a miracle, yet, being continuously, would contradict the nature of the true miracles, which are sporadic interventions of God, in order to supply the order of grace and not that of nature.[37] This is an important point of the discussion, because in a sense it pertains again to the maintenance role of God. For Clarke the denial of God's continuous direct intervention in the natural world would mean in fact somehow God's deprivation of his rights as the supreme ruler, who, as a benevolent monarch should constantly intervene into the affairs of his kingdom.

Yet recall that something is miraculous whether either surpasses the power of created beings or transcends the human knowledge. But, God in his supreme benevolence, would not make physical laws miraculous in this sense, for we have the power to discover them in the order of nature, studying it purely geometrically. This will bring us to another related issue, namely, to the connection between physics and metaphysics. Leibniz confesses about his dissatisfaction with dealing only with the geometrical order of the world:

> There was a time when I believed that all the phenomena of motion could be explained on purely geometrical principles, assuming no metaphysical presuppositions, and that the laws of impact depend only on the composition of motions. But, through more profound meditation, I discovered that this is impossible, and I learned a truth higher than all mechanics, namely, that everything in nature can indeed be explained mechanically, but the principles of mechanics themselves depend on metaphysical and, in a sense, moral principles, that is, on the contemplation of the most perfectly effectual [operans] efficient and final cause, namely, God, and cannot in any way be deduced from the blind composition of motion. And thus, I learned that is impossible for there to be nothing in the world except matter and its variations, as the Epicureans held. (A VI iv, 1976 / AG 245)

Gregory Brown sees here the initiation of the construction of a methodological wall between physics and metaphysics, or, in his terms, the seeds of "the methodological Apartheid between Physics and Metaphysics". I would not go so far in characterizing the relation between the two disciplines, or, better said, between two levels of investigating the universe. I would rather say together with Daniel Garber[38] that, for Leibniz: "mechanistic explanation of the phenomena are preferable not because they are truer than the Scholastics' account, only more

informative. When understood properly, Leibniz's mechanical explanations do not replace Scholastic explanations, but are grounded in them; mechanical explanations are a schematic and partial way of describing what goes in the form, what all forms have in common. It is what we have to fall back on in our ignorance of what specifically God programmed in." Perhaps a Hegelian would say about this that Aristotle's substantial forms were the thesis, Descartes' mechanical philosophy of nature the antithesis, and, eventually, Leibniz' mature conception of the world (of monads) the synthesis. However, I agree with Brown that there is a certain methodological wall between Leibniz and Newton, since the latter says the following:

> I have not yet been able to deduce from phenomena the reason for these properties of gravity, and I do not feign hypotheses. For whatever is not deduced from the phenomena must be called a hypothesis, and hypotheses, whether metaphysical or physical, or based on occult qualities, or mechanical, have no place in experimental philosophy. In this experimental philosophy, propositions are deduced from the phenomena and are made general by induction. The impenetrability, mobility, and impetus of bodies, and the laws of motion and the law of gravity have been found by this method. And it is enough that gravity really exists and acts according to the laws that we have set forth and is sufficient to explain all the motions of the heavenly bodies and of our sea. (CW 943)

Newton defines force solely on the basis of its measurable effects without any reference to its metaphysical origin. Thus, he constructed a systematic account that yields dynamical predictions by defining force in terms of acceleration. On the other hand, Leibniz thought that physics alone could not provide the ultimate knowledge of the world, unless it's based on the more fundamental metaphysical problems. This is an important point of divergence between Leibniz and Newton, and this disagreement concerns both the way they regard the universe as a whole and the way in which we may attain scientific knowledge in such a universe. Yet, as Brown himself observes, the wall between the two is in a sense unidirectional, for Leibniz appreciates also the great benefits of the experimental method in science, since he says in a letter to Abbe Conti:

> I … admire very much the physico-mathematical thoughts of M. Newton, and the public would be greatly in your debt, sir, if you could this capable man to give us his conjunctures in physics to this point. I strongly approve

> his method of deducing from phenomena what can be deduced from them without supposing anything, though this would sometimes be only to deduce conjectural consequences. However, when the data are not sufficient, it is permitted (as is done in deciphering) to conceive hypotheses, and if they succeed, we accept them provisionally, until some new experiences bring us new data, and what Bacon calls a crucial experiment for deciding between the hypotheses. (…)
> I am very much in favor of the experimental philosophy, but M. Newton errs greatly when he supposes that all matter is heavy (or that each part of matter attracts every other part), which experience does not establish, as M. Huygens has already very rightly determined. The gravitating matter cannot itself have this heaviness of which it is the cause, and M. Newton does not furnish any experience or sufficient reason for the void and atoms, or for universal mutual attraction. And since it is not known perfectly and in detail how gravity is produced, or elastic force, or magnetic force etc., we do not on that account have reason to appeal to scholastic occult qualities or to miracles. (TSHT VI, 252-3)

The problem of induction seems to make a wide difference between Newton and Leibniz, for the former it is the true and solely path to attain solid knowledge about the empirical world, whereas for Leibniz, neither senses nor induction provide us secure knowledge about the world. In a letter to Queen Sophia Charlotte of Prusia, he points out that knowledge of universal is possible not because of something outside us, but rather because of something inside, i.e. our innate knowledge:

> [T]here is a light which is born in us. For since the senses and induction can never teach us truths that are fully universal or absolutely necessary, but only what is and what is found in particular examples, and since we nonetheless know the universal and necessary truths of the science – in this we are privileged above the beasts – it follows that we have drawn these truths in part from what is within us. (L 551)

We may say that for Leibniz, as the philosopher Karl Popper will say three hundred years later, the experiments are conducted to test out hypotheses and not to make pure discoveries through the mere study of the surrounding world. When conducing an experiment, we approach the reality with the help of a certain supposition which requires confirmation in order to be further used as a piece of knowledge. This is in fact why very rarely Leibniz uses his physics to discuss or explore metaphysics, for tone is always one of confirmation and never of justification.

Later on, Leibniz becomes clear about the metaphysical differences between considering these two kinds of interaction: body-body, through action at a distance, and mind-body, through pre-established harmony:

> There is no comparison between the action of one body on another and the influence of the soul on the body. There is immediate contact between bodies, and we understand how that can be, and how, since there is no penetration, their coming together must alter their movement in some way. But we see no such consequences with the soul and the body: these two do not touch, and do not interfere with one another in an immediate way which we can understand and deduce from their natures. All they can do is to be in agreement, and to depend on one another by a metaphysical influence, so to speak, in virtue of the soul's ideas; and that is contained in God's plans, which are in conformity with them. They are related through the mediation of God, not by a continual interruption of the laws of the one because of its relation to the other, by a harmony pre-established in their natures *une fois pour toutes*. (G VI 570)

Interestingly, as Brown has also pointed out, there is also another reason for rejecting the universal gravitational force and this goes together with the rejection of a vacuum.

In *Anti-barbarus physicus* Leibniz says that the defenders of gravity "are forced by this view of the essential attraction of matter to defend the vacuum, since the attraction of everything for everything else would be pointless if everything were full. But in the true philosophy, the vacuum is rejected for other reasons." (G VII 343 / AG 318)

For Leibniz cannot be any vacuum in the actual world, because it is the best possible world and thus the space of that vacuum could have been filled with another finite substance, thus increasing the perfection of the world even further, but since it is the best possible world, its perfection is maximal so there is no vacuum in our universe. Moreover, the existence of void would violate Leibniz's identity of indiscernables, for in the vacuum it would have been possible to have again two identical portions of space. But thus again violates the maximal perfection of the actual world and ultimately the great principle of sufficient reason. There is no reason for a benevolent God to create two identical things in the most perfect world, because a world with more diversity, i.e. no identical items in it, would have been more perfect; therefore the perfection of the best possible world requires a plenum.

Note that this metaphysical rejection of vacuum has other important physical consequences, namely the rejection of universal attraction between bodies and the rejection of absolute space, for both would require the existence of void.

IV. Leibniz on *Matter* and *Force*. Physics and Metaphysiscs

Perhaps the central methodological issue which marks the transition from Scholastic philosophy to modern philosophy of nature in the seventeenth century is the continuous strive to explain mechanically the universe. Central to this physical picture was the domain of dynamics. As David Papineau observes:

> What is relevant is that something like the 'principle of inertia' was generally assumed, in that it was accepted that a body would continue in any motion, with the same speed in the same direction, until subject to some external action. (…)
> As a result, the laws governing the changes of motion which result from impacts were considered to be of fundamental importance in the seventeenth century. It was in the problem of deriving these laws that the motion of 'force of motion' had its most important application.[39]

One of the first who tried to provide a systematic account of collision and its conservation of the 'force of motion' was Descartes. The problem was that he considered the forces of bodies implicated in impact is proportional to its 'quantity of motion' (mass times speed), a conception which was soon attacked and proved to be wrong by physicists like C. Hughes, C. Wren or J. Wallis. In the light of these rejections of the Cartesian framework for collisions between bodies, Leibniz, on one hand, agreed that it is something wrong with this conservation principle, but, on the other hand, he was reluctant to banish from the mechanical explanation of the world. Why and to what cost? Well, once again, metaphysics seems to play a fundamental and foundational role in his system. He saw that something is wrong with Descartes' dynamics,[40] yet he could not rule out the existence of a perfect universe where the total sum of 'forces of motion' should be conserved and thus this perfect machinery does not need any supplementation or further adjustment (from its Creator).[41] Thus, Leibniz was metaphysically compelled to come with

something new, with a physical quantity which can be said to be conserved in spite of the various collisions in the universe. What is conserved in impact according to Leibniz is the 'living force' (*vis viva*) of the bodies, a 'force' which is given by mass times squared velocity. In his fifth letter to Clarke, Leibniz says:

> The author objects, that two soft or un-elastic bodies meeting together, lose some of their force. I answer, no. 'Tis true, their wholes lose it with respect to their total motion; but their parts receive it, being shaken (internally) by the force of the concource. And therefore that loss of force is only in appearance. (G VII 414: A87-8)

This was so, because Clarke defended Newton, who, along with some Cartesians, maintained that mass times velocity, taken as a vector quantity, should be considered as the appropriate measure of the 'force' of a moving body.[42]

But, as Leibniz would acknowledge soon, the essence of matter does not consist in extension alone, and thus the purely mechanical explanation of the natural universe should be supplemented with deeper metaphysical understanding of the world. In *Critical Thoughts on the general part of Descartes' Principles* (1692), assessing Descartes' *Principles*, Leibniz says explicitly:

> [Descartes] says, namely, that no other principles are necessary for the explanation of natural phenomena than those taken from abstract mathematics, or from the doctrine of size, figure, and motion, and that he recognizes no other matter than that which is the subject to geometry. I fully agree that all the particular phenomena of nature can be explained mechanically if we explore them enough and that we cannot understand the causes of material things on any other basis. But I hold that, nevertheless, that we must also consider how these mechanical principles and general laws of nature themselves arise from higher principles and cannot be explained by quantitative and geometrical considerations alone; that is rather something metaphysical in them, which is independent of the concepts which imagination offers, and which it to be referred to a substance devoid of extension. For besides extension and its variation, there is in matter a force or power of action by which the transition is made from Metaphysics to nature, from material to immaterial things. This force has its own Laws, which are deduced from the principles, not merely of absolute, and so to speak brute necessity, but of perfect reason. (GP iv391: L675/409)

Thus, as Margaret Wilson says:

> Fundamentally, Leibniz was concerned to oppose – for religious reasons especially – the 'geometrical' conception of natural sciences exemplified (in different degrees) by his predecessors Descartes and Spinoza. This is to say, he was concerned to oppose the assimilation of physics to geometry, and of physical necessity to geometrical necessity.[43]

This Cartesian characterization of physics as nothing but geometry comes naturally with a close connection between strict Determinism and the conception of matter as extension. But Leibniz was equally opposed to both, striving how to accommodate both substantial forms and mechanical principles, and, on a different level, human freedom with necessary laws of nature. Some extracts from his works could exemplify this point:

> If mechanical rules depended on Geometry alone without metaphysics, phenomena would be quite different. (GP iv. 446 / LG 36)
> I have already asserted several times that the origin of mechanism itself does not spring from a material principle alone and mathematical reasons but from a certain higher and so to speak Metaphysical source. (…)
> One remarkable proof of this, among others, is that the foundations of the laws of nature must be sought not in this, that the same quantity of motion is conserved, as was commonly believed, but rather in this, that is necessary that the same quantity of active power be conserved. (GP iv.505-6: L 811/499)
> Perhaps someone will (…) believe that a completely geometric demonstration can be given of [the laws of motion], but in another discourse I will show that the contrary is the case, and demonstrate that they cannot be derived from their source without assuming architectonic reasons. (GP vii.279: L 788/484)

These architectonic reasons are metaphysical principles with whom one could both ground and better understand the mechanical/dynamical/geometrical laws of nature.

> Thus, since the wisdom of God has always been recognized in the detail of mechanical structure of some particular bodies, it ought also to show itself in the general economy of the world and in the constitution of the laws of nature. And this is so true that one notices the counsels of his wisdom in the laws of motion in general. (G IV 466)

Or, as he acknowledges in *Discourse of Metaphysics* (1686): "And it becomes more and more apparent, although all particular all particular phenomena can be explained mathematically or mechanically by those who understand them, that nevertheless the general principles of corporeal nature and of mechanics itself are rather metaphysical than geometrical and belong rather to some indivisible forms or natures as causes of appearances than to corporeal or extended mass." (GP iv.444: LG32) The conclusion is clear and it looks like the one he drew in *On the Elements of Natural Science* (ca.1682-4): certain things take place in a body which cannot be explained by the necessity of matter alone. Such are laws of motion which depend on the metaphysical principle of the equality of cause and effect. Therefore we must deal here with the soul, and show that all things are animated. (L 429/278)

Summing up the whole discussion, I think that we can say now that, in order to understand Leibniz's systematic account of the world, we must acknowledge three interconnected levels of analysis:

- the metaphysical level of monads as the ultimately real level of reality;
- the physical level of phenomena, which are grounded on monads and their states;
- the mathematical level of ideal entities, which *qua ens rationis* (numbers, infinitesimals, space, time…) are 'useful fictions' which could help us better understand the gap between the first two levels of analysis of the universe.[44] Thus, both implicitly and explicitly, the connection and the foundational role played by each become apparent on this scheme.

I will conclude by showing how actual seems to be the Lebnizian analysis and how a contemporary physicist think the connections do operate between the three levels of reality. I will exemplify this providing the interesting and provocative account of Carl von Weizsäcker, who, I his book *The World View of Physics*, says that:

> Probably no philosopher has penetrated metaphysics and the mathematical sciences with so unified a movement of thought as Leibniz. What he thus achieved in metaphysics with respect to clarity, he certainly did not sacrifice in profundity. But above all he has thus placed the mathematical sciences before the background appropriate to them. To follow his thoughts through is to perceive how, in the choice of our simplest mathematical concepts, we make metaphysical decisions.[45]

And he continues his analysis as follows:

The argument of the Leibnizian *Theodicy* is exactly analogous to that of the variational principles of physics. Let us take as perhaps the most familiar example Fermat's principle of the shortest path of light. This principle declares: A ray of light chooses that path on which it travels the distance from its initial point to its end point in the shortest possible time. The three basic laws of geometrical optics can be derived from this principle: the law of the rectilinear propagation of light and the laws of reflection and refraction.[46]

And he continues further by saying that:

Thus they [possible worlds] are exactly like the possible paths of light, merely imaginary things, which are constructed for a methodological purpose. (…)
This comparison loses its somewhat playful tone, if we consider that the variational principles of physics were of the optimal character of the real world. In the best possible world variational principles must be valid, and that such principles are valid in the real world confirms the fact that it is the best. The shortest path of light is the best, and a world in which light did not choose the shortest path could therefore not be the best.[47]

Thus, the conclusion comes naturally that:

The variational principles have often been understood to be the expression of a finalistic, purposive character of cosmic events. In general the laws of nature are valid for us as the expression of causality reigning throughout nature, according to which the state of a thing and its environment at any instant determines the state of the thing in the instant immediately following. The variational principles appear to break through this schema by means of a teleological type of order: the path of light is determined through the end point which the light ray is to reach only after this path has been traversed. The factor determining the event seems here to be not a 'mechanical' cause, but a goal to be reached in the future, a purpose. So it was concluded that there is a plan governing nature, and thus a spiritual principle in nature.[48]

So, the final conclusion is that :

> God does not need the world for some use – but he wants it to be perfect. And the perfection of a world in which variational principles hold, consists in the fact that it unites the greatest richness in phenomena by a law as simple, and as transparent for the mind as possible; it consists in the fact that such a world possesses the greatest intellectually beauty.[49]

It is remarkable how a contemporary hardcore physicist could still think of the world in Leibniz's terms, proving thus unequivocally still the contemporary fruitfulness and percipience of the Leibnizian ideas.

NOTES

1 As for my opinion, I have said more than once that I hold space to be something merely relative, as time is, that I hold it to be an order of coexistences, as time is an order of successions. (L III 4)

2 One may say that none of this is entirely true and that in fact some theological problems concerning the reconciliation between Roman Catholics and Protestants motivated Leibniz's work, and issues as *transubstantiation* or *incarnation* lay in fact the foundations of his metaphysics. Of course that this interpretation is at least partially true, but exceeds the limits of this work and thus it will not pursue it here.

3 B. Russell, Couturat; plus partially N. Rescher and B. Mates.

4 H. Scholz.

5 D. Garber.

6 It may be noted that in fact PII is derivative from PSR, in the sense that even though God could have been created two identical things in the world, there is no reason to act in such manner, since the presence of two identical items would decrease the amount of perfection of the world, yet the actual world is the best possible world, and thus the most perfect one.

7 In the case of the latter, it would be better to say that in fact it concerns the relation between physics and metaphysics.

8 Vailati (1997), p. 122.

9 See Vailati (1997), Broad (1946), Hartz and Cover (1988) and last but not least Alexander (1956) in LC

10 Leibniz simply failed to accept what later will be seen exactly as the central feature of the infinite- when dealing with infinite quantities, in some sense, the part and the whole can be equal"; Mancosu (1996), p. 129.

11 "Definition. A system S is said to be *infinite* when it is similar to a proper part of itself; in the contrary case S is said to be a *finite* system"; Dedekind (1963), p.63.

12 "Two different aggregates have the same cardinal number when and only when they are what I call 'equivalent' to one another, and there is no contradiction when, as often happens with infinite aggregates, two aggregates of which one is a part of the other have the same cardinal number. I regard the non-recognition of this fact as the principal obstacle to the introduction of infinite numbers"; Cantor (1955), p.75.

13 "The concept of totality is in this case simply the representation of the completed synthesis of its parts; for, since we cannot obtain the concept from the intuition of the whole— that being in this case impossible— we can apprehend it only through the synthesis of the parts viewed as carried, at least in idea, to the completion of the infinite"; Kant (1965), p. 398.

14 For Leibniz, one of the reasons for rejecting such a number as being 'natural' is the impossibility of saying whether in that case this number is even or odd, feature that can be easily determined in other cases of real natural numbers.

On the other hand, Cantor identifies this point as an obvious mistake: "All so-called proofs of the impossibility of actually infinite numbers are, as may be shown in every particular case and also on general grounds, false in that they begin by attributing to the numbers in question all the properties of finite numbers, whereas the infinite numbers, if they are to be thinkable in any form, must constitute quite a new kind of number as opposed to the finite numbers, and the nature of this new kind of number is independent on the nature of things and is an object of investigation, but not of our arbitrariness or our prejudice"; Cantor (1955), p. 74.

15 Leibniz is rejecting both infinite collections (for not being true *wholes*) and infinite numbers, (for being contradictory elements), even though they are different problems. The mathematical bridge between them seems to be the magnitude of the collection understood as the cardinal number assigned to that collection.

16 "Leibniz had two quite different accounts of the infinite in play. His constructist stance in mathematics led him to treat the mathematical infinite as merely *potentially* infinite, whereas his metaphysics of divided matter led him to treat the division in bodies as *actually* infinite"; Brown (2000), p. 36.

17 "The multitude of things, he (Leibniz) says, passes every finite number, or rather every number. It cannot be denied that this position is consistent with his principles, and is even a direct result of them. But the consistency is of that kind which shows a mistake in the principles"; Russell (1992), p. 117.

18 A clear and succinct characterization of the distinction can be found in Moore (1990), p. 51; "Roughly: to use 'infinite' *categorematically* is to say that there is something which has a property that surpasses any finite measure; to use it *syncategorematically* is to say that, given any finite measure, there is something which has a property that surpasses it".

19 Arthur (2001), p. 46.

20 I disagree with Arthur concerning his way of dealing with Cantorian transfinite numbers. He rejects Cantor's idea of the first infinite ordinal ?, on the basis that ?-1 is neither finite, nor infinite. But Cantor is quite explicit with this kind of things: "I call ? the limit of the increasing, finite, whole numbers ?, because ? is the least of all numbers which are greater than all the finite numbers. But ?-? is always equal to ?, and therefore we cannot say that the increasing numbers ? come as near as we wish to ?; indeed any number ? however great is quite as far off from ? as the least finite number. Here we see especially clearly the very important fact that my least transfinite ordinal number ?, and consequently all greater ordinal numbers, lie quite outside the endless series 1,2,3 and so on. Thus ? is not a maximum of the finite numbers, for there is no such thing"; Cantor (1955), p. 78.

21 Carling (1997), p.10.

22 "For Aristotle the infinite was the *untraversable*"; Moore (1990), p. 40.

23 Moore (1990), p. 79.

24 "Imagine a glass jar halfway full of metal coins, say copper pennies. Resting squarely on the bottom of the jar, one coin is uniquely the largest of them all: this is the *alpha penny*. Otherwise like an ordinary penny, the alpha penny is one-half inch thick. Also inside the jar there is a penny, just one, which, like all the pennies in the jar, is ordinary in its width and circumference but which is exactly half as thick as the alpha penny; this second penny measures one-quarter inch in thickness. (…) And so on, ad infinitum. (…) (There is no 'omega penny', so to speak, of only infinitesimal thickness, to be found anywhere in the jar". Levey (1999), p. 143.

25 "Here's a block of stone one cubic foot in volume abd regular in its dimensions: it measures one foot high by one foot long by one foot wide. On close inspection it is observed that the block is neatly divided down the middle by a hairline fisure into two equals slabs, each one foot high by one foot long by only half a foot wide. (…) And so on ad infinitum. (…) Each an every slabs is seen, in fact, to give way to two smaller slabs, without end". Levey (1999), p. 144.

26 Even though Levey's analysis is very interesting and challenging, I do not want to present it here in detail, just because I do not think that may shed any important light to our main problem of the relation between mathematics and metaphysics in Leibniz's philosophy of infinite. On the other hand, I recognize that the original Leibnizian model might cause some troubles, but I think that they could be solved by a critical discussion concerning continuous & discrete objects; I will discuss this point in the next part.

27 From *Pacidius Philalethei (1676)*, quoted in Levey (1999), p. 144.

28 "The activities or the acts of God are commonly divided into ordinary and extraordinary. But it is well to bear in mind that God does nothing out of order. Therefore, that which passes for extraordinary is so only with regard to a particular order established among the created things, for as regards the universal order, everything conforms to it. This is so true that not only does nothing occur in this world which is absolutely irregular, but it is even impossible to conceive of such an occurrence" (DM VI)

29 Incarnation could be seen also as an argument, beside the usual one (the best result regarding the simplicity of hypotheses and the richness in phenomena) that we are living in the best possible world. this is so just because God sent his son in this particular world and not in other, which means that this world has to have something special feature in comparison with the others.

30 See Adams (1994), p.92

31 Not underlined in the original text.

32 [I]n the philosophical sense [a miracle is] that which exceeds the power of created beings. (G IV 520 / L 494)

33 [A] miracle is a divine action which transcends human knowledge; or more strictly which transcends the knowledge of creatures, or [vel] in which God acts beyond the order of nature. (C 508)

34 If a miracle differs from what is natural only in appearance and with respect to us, so that we call that only a miracle which we seldom see, there will be no internal real difference between a miracle and what is natural, and at the bottom everything will be either equally natural or equally miraculous. Will divines like former, or philosophers the latter? (L V 110)

35 von Weizsäcker (1952), p.184.

36 Brown (2007), p.147.

37 According to my opinion, the same force and vigour remains always in the world, and only passes from one matter to another, agreeably to the laws of nature, and the beautiful pre-established order. And I hold, that when God works miracles, he does not do it in order to supply the wants of nature, but those of grace. Whoever thinks otherwise, must need have a very mean notion of the wisdom of God. (L I 4)

38 Garber (1985), p.99.

39 Papineau (1977), p. 142.

40 M. Descartes' physics has a great defect; it is that his rules of motion are for the most part false. This is demonstrated. And his great principle, that the same quantity of motion is conserved in the world, is an error. What I say here is acknowledged by the ablest people in France and England. Letter to Filippi (G IV 286)

41 It conforms to reason to say that the same sum of motive force is conserved in nature; this sum does not diminish, since we never observe any body lose any force that is not transferred to another; nor does this sum increase since perpetual motion is unreal to such a degree that no machine and in no consequence not even the entire world can conserve its force without new impulsion from without. (GM VI 117: L456/296)

42 For more of this controversy see Papineau (1977).

43 Wilson (1976), p.122.

44 This picture is not new and it was endorsed by Winterbourne (1982) and Hartz & Cover (1988), and partially by Rescher (1979) and Mates (1986).

45 von Weizsäcker (1952), p. 182.

46 idem, p.186.

47 idem, p.188.

48 idem, pp.188-9.

49 idem, pp. 192-3.

BIBLIOGRAPHY

Primary Sources:

LEIBNIZ:
A = *G.W.Leibniz. Sämtliche Schriften und Briefe*, Akademie Verlag
AG = G.W.Leibniz. Philosophical Essays, Ariew, R. & Garber, D. (eds) (1989)
C = *Opuscules et Fragments inedits de Leibniz*, Couturat, L. (ed.)
G = *Die Philosophische Schriften* von G.W. Leibniz, 7 vols., Gerhardt, C.I. (ed.)
GM = *Die Mathematische Schriften* von G.W. Leibniz, 7 vols., Gerhardt, C.I. (ed.)
L = G.W. Loemker: Philosophical Papers and Letters, Loemker (ed.)
LC = *The Leibniz-Clarke Correspondence*, Alexander, H.G. (ed.) (1956)
T = *Theodicy*, Farrar, A (ed.) (1985)

NEWTON:
CW = I. Newton *The Principia: Mathematical Principles of Natural Philosophy*, Cohen, I.B. & Whitman, A. (1999), University of California Press
TSHT = *The Correspondence of Isaac Newton*, 7vols. Turnbull, H.W. & Scott, J.F. &Tilling, L. (1959-77), Cambridge University Press

Secondary Sources:

Adams, R. (1994) *Leibniz: Determinist, Theist, Idealist*, Oxford University Press
Alexander, H.G. (ed.) (1956) *The Leibniz-Clarke Correspondence*, Manchester University Press
Arriew, R. & Garber, D (1989) *G.W. Leibniz: Philosophical Essays*, Hackett Publishing Co
Arthur, R. (1998) "Infinite Aggregates and Phenomenal Wholes: Leibniz's Theory of Substance as a Solution to the Continuum Problem", in *Leibniz Society Review*, vol. 8, p. 25-45
---------- (1999) "Infinite Number and the World Soul; in Defence of Carlin and Leibniz", in *Leibniz Society Review*, vol. 9, p.105-116
---------- (2001) "Leibniz and Cantor on the Actual Infinite", in Poser, H. (ed.) Proceedings of the VII International Leibniz Congress, p. 41-46
---------- (2001a) *The Labyrinth of the Continuum.Leibniz's Writings on the Continuum Problem*, 1672-1686; translated, edited and with and introduction by Richard Arthur, Yale University Press
Bassler, B. (1998) "Leibniz on the Indefinite as Infinite", in *The Review of Metaphysics* 51, p. 849-874
Bell, J. (1998) *A Primer of Infinitesimal Analysis*, Cambridge University Press
--------- (2001) *The Art of Inteligible. An Elementary Survey of mathematics in its Conceptual Development*, Kluwer Academic Publishers
Benardete, J. (1964) *Infinity. An Essay in Metaphysics*, Clarendon Press, Oxford

Briedges, D. & Richman, F. (1987) (eds.) *Varieties of Constructive Mathematics*, Cambridge University Press
Broad, C. (1975) *Leibniz: An Introduction*, Cambridge University Press
Broad, C.D. (1946) "Leibniz's Last Controversy with the Newtonians", in Woolhouse (1994), pp. 1-19
Brouwer, L.E.J. (1975) *Collected Works*, vol.I, edited by A. Heyting; North-Holland Publishing Company
Brown, G. (1998) "Who's Afraid of Infinite Numbers? Leibniz and the World Soul", in *Leibniz Society Review*, vol. 8, 113-125
---------- (2000) "Leibniz on Wholes, Unities and Infinite Numbers", in *Leibniz Society Review*, vol. 10, p. 21-51
---------- (2007) "Is the logic in London different from the logic in Hanover?. Some methodological issues in Leibniz's dispute with the Newtonians over the cause of gravity", in Phemister, & Brown (2007), pp.145-162
Cantor, G. (1955) *Contributions to the Founding of the Theory of Transfinite Numbers*, Dover.
Carlin, L. (1997) "Infinite Accumulations and Pantheistic Implications: Leibniz and the Anima Mundi", in *Leibniz Society Review*, vol. 7, p. 1-24
Dedekind, R. (1963) *Essays on the Theory of Numbers*; Dover
Dummett, M. (1977) *Elements of Intuitionism*, Clarendon Press
Flew, A. (1967) "Miracles", in Edwards, P. (ed.) *Encyclopedia of Philosophy*, MacMillan and Free Press
Frege, G. (1996) *The Foundations of Arithmetic*, Northwestern University Press.
Garber Garber, D. (1985) "Leibniz and the Foundations of Physics: The Middle Years", in Okruhlik & Brown (1985), p. 95
Harz, G.A. & Cover, J.A. (1988) "Space and Time in the Leibizian Metaphysics", in Woolhouse (1994), pp. 76-103
Kant, I. (1965) *Critique of Pure Reason*, St Martin's Press
Ishiguro, H. (1990) *Leibniz's Philosophy of Logic and Language*
Larmer, R. (1996) "Miracles and the Laws of Nature", in Larmer, R.A. *Questions on Miracles*, McGill-Queen's University Press
Levey, S. (1998) "Leibniz on Mathematics and the Actual Infinite Division of Matter", in The Philosophical Review, vol. 107, No.1. p. 49-96
---------- (1999) "Leibniz's Constructivism and Infinitely Folded Matter", in Gennaro, R & Huenemann, C. (eds.) *New Essays on the Rationalists*; Oxford University Press
Levine, M.P. (1996) "Miracles", in *Stanford Encyclopedia of Philosophy*, web publication
Mancosu, P. (1996) *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century*, Oxford University Press
Mates, B. (1986) The *Philosophy of Leibniz. Metaphysics and Language*, Oxford University Press
McRae, R. (1985) "Miracles and Laws", in Okruhlik, K. & Brown, J.R. (1985)

Moore, A. (1990) *The Infinite*; Routledge
Okruhlik, K. & Brown, J.R. (eds.), (1985) *The Natural Philosophy of Leibniz*, D. Reidel Publishing Company
Papineau, D. (1977) "The Vis Viva Controversy", in Woolhouse (1981), pp.139-56
Phemister, P. & Brown, S. (eds.) (2007) *Leibniz and the English-speaking world*, Springer
Rescher, N. (1967) *The Philosophy of Leibniz*, Englewood Cliffs
Rescher, N. (1979) *Leibniz: An Introduction to his Philosophy*, Oxford University Press
Rescher, N. (1981) *Leibniz's Metaphysics of Nature: A Group of Essays*, Dordrecht
Russell, B. (1992) *The Philosophy of Leibniz*, Routledge
Sleigh, R.C. (1990) *Leibniz & Arnauld: A Commentary on their Correspondence*, Yale University Press
Troelstra, A.S. & Van Dalen, D. (1988) *Constructivism in Mathematics*, vol. I; North-Holland
Vailati, E. (1997) *Leibniz & Clarke. A Study of Their Correspondence*, Oxford University Press
von Weizsäcker, C. (1952) *The World View of Physics*, Routledge and Kegan Paul
Wilson, M. (1976) "Leibniz's Dynamics and Contingency in Nature", in Woolhouse (1981), pp.119-38
Woolhouse, R.S. (ed.) (1981) *Leibniz: Metaphysics and Philosophy of Science*, Oxford University Press
Woolhouse, R.S. (ed.) (1994) *G.W. Leibniz. Critical Assessments*, vol. III, Routledge
Woolhouse, R.S. & Franks, R. (1998) *G.W. Leibniz. Philophical Texts*, Oxford University Press